Coverage and quality driven training of generative image models

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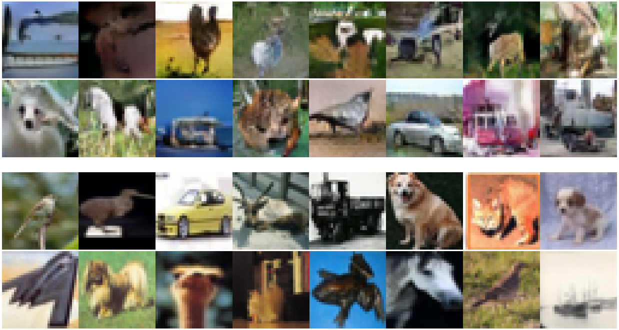
December 2018

What are generative (image) models?

1. Density estimator $p(x)$ from which we can sample

2. Models that generalize outside train data



CIFAR-10: 32×32 samples from model and training set

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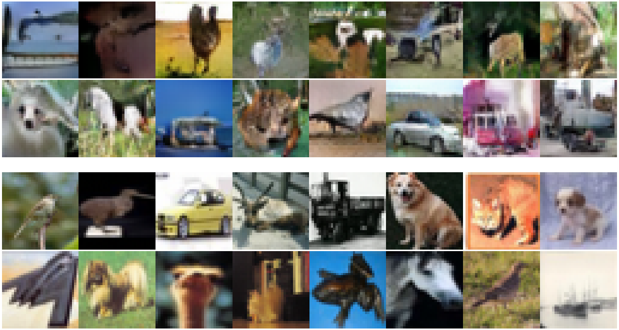
2. Models that generalize outside train data
3. Models that allow to assess if (2) actually happened !



CIFAR-10: 32×32 samples from model and training set

Generative image models - motivation

- Sand-box problem to study complex density estimation
  - Images: high-dimensional non-trivial distributions
- Conditional generative models are useful in practice
  - Generate image, speech,. . . conditioned on attributes, text, . . .
  - Conditioning on some input is the "easy" part
- Representation learning from unlabeled data
  - Leveraging latent variables and/or internal feature maps

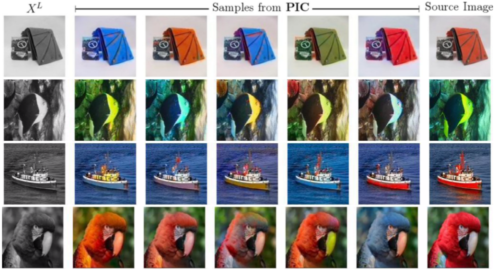


Image colorization results form [Royer et al., 2017]

Current approaches in deep generative modeling

- Autoregressive models, e.g. Pixel-CNN [Oord et al., 2016]
  - Model conditionals in $p(x) = \prod_i p(x_i | x_{<i})$, *e.g.* with CNN, RNN,...
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- Variational inference [Kingma and Welling, 2014, Rezende et al., 2014]
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  - VAE: Decoder $p_\theta(x|z)$, encoder $q_\phi(z|x) \approx p(z|x)$

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- Generative adversarial networks [Goodfellow et al., 2014]
  - Deterministic $x = G_\theta(z)$, low dim. support, likelihood-free
  - Use discriminator real/synth. samples as "trainable loss"

Adversarial and maximum likelihood training

- Discriminator in GAN trained with binary cross-entropy loss

$$\mathbb{E}_{p_{\text{train}}(x)}[\ln D(x)] + \mathbb{E}_{p_\theta(x)}\left[\ln\left(1 - D(x)\right)\right] \tag{1}$$

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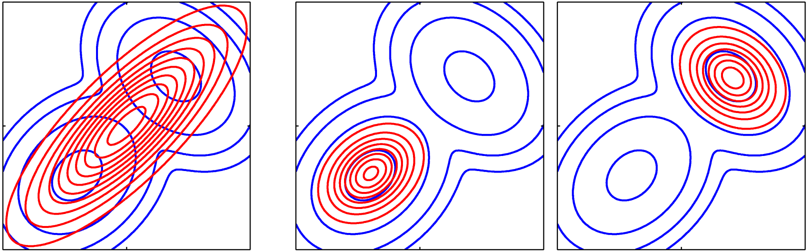
- Adding a constant we obtain $\mathcal{L}_C(\theta) - H(p_{\text{train}}) = D_{\text{KL}}(p_{\text{train}} || p_\theta)$

Mode dropping and over-generalization

- Reversing KL direction yields qualitatively different estimators
  - [Bishop, 2006]: "zero avoiding" or "zero forcing" behavior



$D_{\mathsf{KL}}(p||q)$ $D_{\mathsf{KL}}(q||p)$

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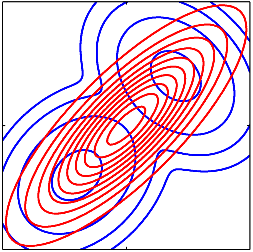

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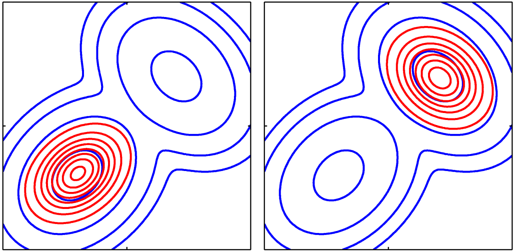

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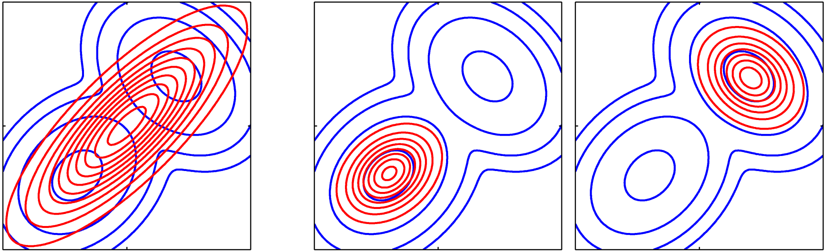


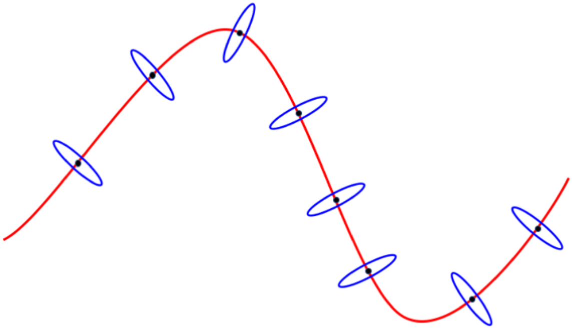
$D_{\mathrm{KL}}(p||q)$
Expectation propagation

$D_{\mathrm{KL}}(q||p)$
Variational inference

Limitation of maximum likelihood estimation

- Only measures the mass on the train data, invariant to where the rest of the mass goes

$$\mathcal{L}_C(\theta) = -\mathbb{E}_{p_{\text{train}}(x)}[\ln p_\theta(x)] \approx \frac{1}{n}\sum_{i=1}^{n} \ln p_\theta(x_i) \qquad (4)$$



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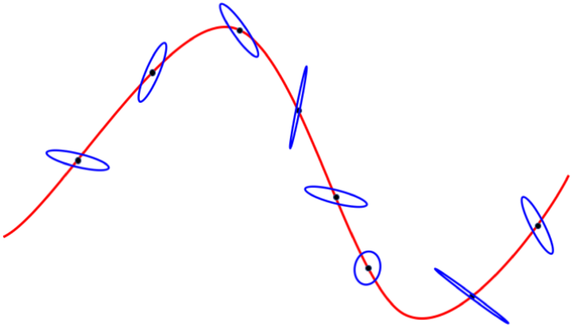

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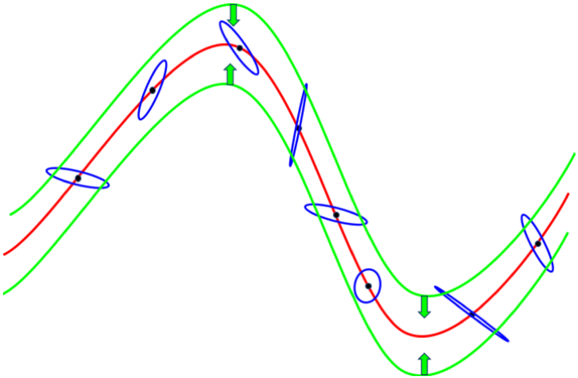
- Idea 1: Use adversarial discriminator to break this invariance
  - Unlike MLE, discriminator is sensitive to model samples



Limitation of basic VAE decoders

- "Vanilla" VAE decoders factorize over data x given latent z

$$p(x|z) = \mathcal{N}\left(x; \mu(z), \mathrm{diag}\left(\sigma(z)\right)\right) \tag{5}$$

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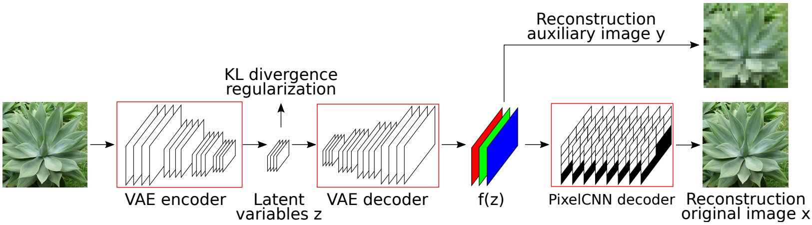

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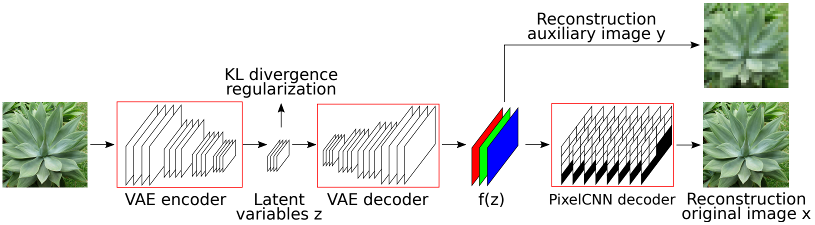
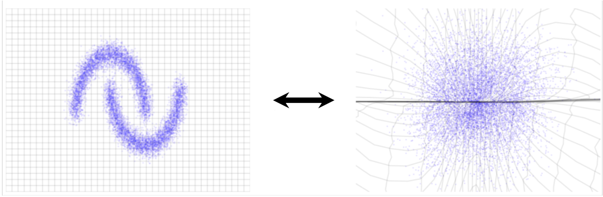
  - Too slow to sample sequential pixelCNN during training



Idea 2: Use NVP in a VAE decoder

Data space $\mathcal{X}$ Latent space $\mathcal{Y}$



- Invertible transformation between image x and feature y

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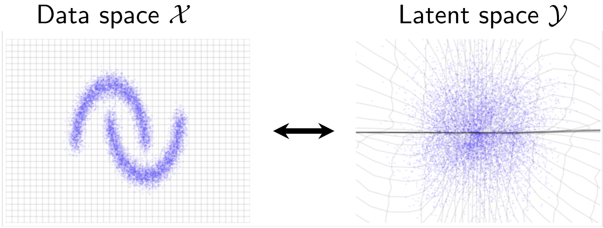

- Invertible transformation between image x and feature y
- Factorized Gaussian decoder over feature y given z

$$p_y(y|z) = \mathcal{N}\left(y; \mu(z), \mathrm{diag}\left(\sigma(z)\right)\right) \tag{6}$$

$$x = f^{-1}(y) \tag{7}$$

$$p_x(x|z) = p_y(f(x)|z) \times \left|\det\left(\frac{\partial f(x)}{\partial x^\top}\right)\right| \tag{8}$$

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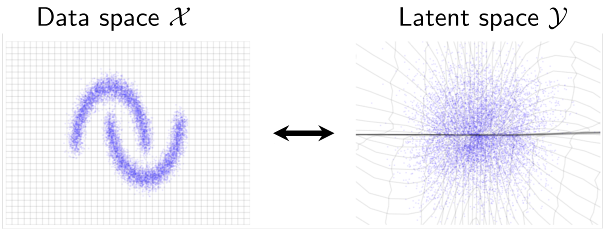

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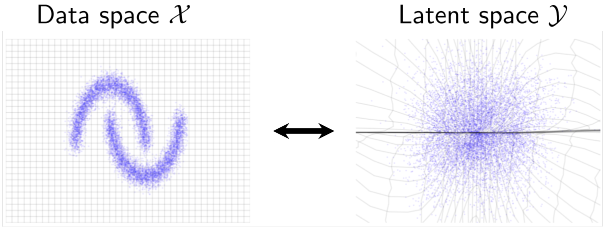

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- Non-factorial non-Gaussian conditional distribution $p(x|z)$
  - Better samples & likelihoods

NVP in a nutshell

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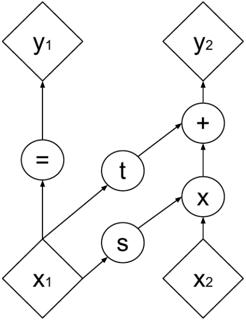

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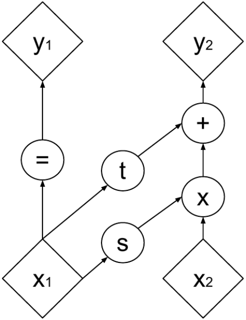

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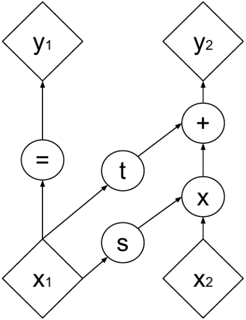

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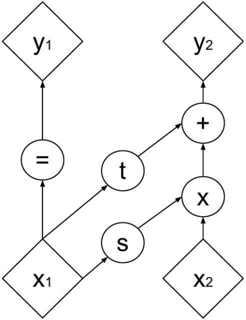

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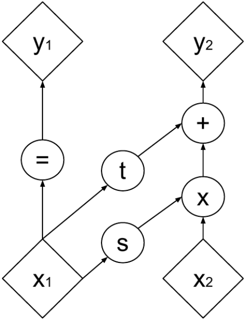
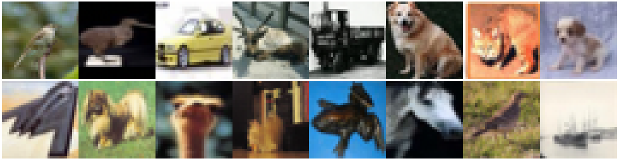


- Triangular Jacobian, determinant is $\operatorname{prod}(s(x_1))$
- Functions $s(\cdot)$ and $t(\cdot)$ may be non-invertible, *e.g.* CNN

Experimental setup



- Focus here on experiments on CIFAR-10 32×32
  - Also quant. + qual. evaluation on STL-10, CelebA, ImageNet, LSUN-bedrooms
- Evaluation metrics
  - Bits per dimension (i.e. negative log-likelihood)
  - Inception score [Salimans et al., 2016]: images should have low label-entropy, and high marginal label entropy
  - Fréchet inception distance [Heusel et al., 2017]: distance real and sampled images in 1st and 2nd moments CNN features

Impact of training objectives and NVP decoder

VAE

CIFAR-10

	$\mathcal{L}_Q$	$\mathcal{L}_C$	NVP	BPD ↓	IS ↑	FID ↓
VAE		✓		4.4	2.0	171.0
GAN	✓			7.0 *	6.8	31.4

*: Obtained using VAE with frozen GAN decoder

GAN

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CQ	✓	✓		4.4	5.1	58.6
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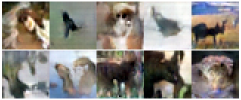

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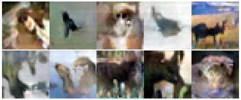

CQ


CQ-F


GAN

Evaluation of more advanced architectures

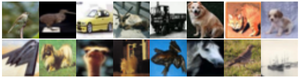
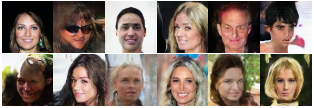
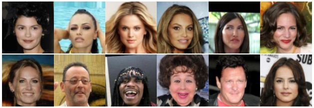
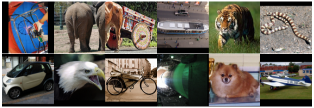
CIFAR-10

	IAF	Residual	BPD $\downarrow$	IS $\uparrow$	FID $\downarrow$
GAN			7.0 (*)	6.8	31.4
GAN		✓	—	7.4	24.0
CQF			3.90	7.1	28.0
CQF		✓	3.84	7.5	26.0
CQF	✓	✓	3.77	7.9	20.1
CQF (large Discr.)	✓	✓	**3.74**	**8.1**	**18.6**

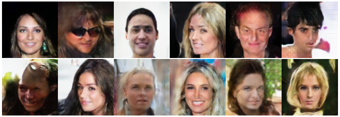
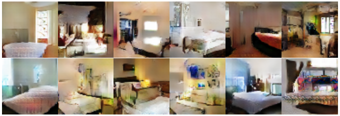
Comparison to the state of the art

	CIFAR-10			STL-10		
	BPD ↓	IS ↑	FID ↓	BPD ↓	IS ↑	FID ↓
DCGAN [Radford et al., 2016]		6.6				
SNGAN [Miyato et al., 2018]		7.4	29.3		8.3	53.1
SNGAN-Hinge [Miyato et al., 2018]					8.7	47.5
BatchGAN [Lucas et al., 2018]		7.5	23.7		8.7	51
WGAN-GP [Gulrajani et al., 2017a]		7.9				
Improved Training GAN [Salimans et al., 2016]		8.1				
SNGAN-ResNet-Hinge [Miyato et al., 2018]		8.2	21.7		**9.1**	**40.1**
Prog-GAN [Karras et al., 2018]		**8.8**				
NVP [Dinh et al., 2017]	3.49					
VAE-IAF [Kingma et al., 2016b]	3.11					
PixelRNN [van den Oord et al., 2016]	3.00					
PixelCNN++ [Salimans et al., 2017]	**2.92**					
CQF [+Residual, +flow, +large D] (Ours)	3.74	8.1	**18.6**	4.00	8.6	52.7
CQF [+Residual, +flow, +2 scales] (Ours)	3.48	6.9	28.9	**3.82**	8.6	52.1

Unconditional CQF samples

	Generated	Training
CIFAR-10		
Celeb-A		
LSUN-bedrooms		
STL-10		
ImageNet-64		

Comparison to NVP samples

	Our samples	NVP samples
CIFAR-10		
Celeb-A		
LSUN-bedrooms		
ImageNet-64		

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- First systematic joint BPD and (IS, FID) evaluation on 7 datasets, results competitive with the state of the art
- Main message: We can have generative models with full support, *and* high quality samples

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