Fisher Vector image representation

Machine Learning and Category Representation 2013-2014

Jakob Verbeek, December 13, 2013

Course website:

http://lear.inrialpes.fr/~verbeek/MLCR.13.14

Fisher vector image representation

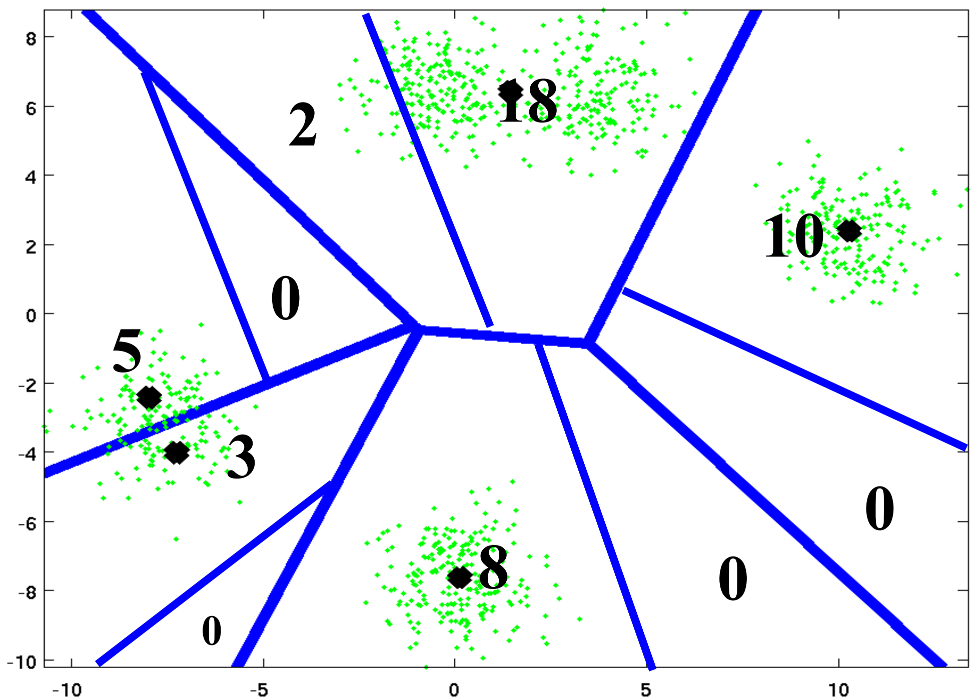
- An alternative to bag-of-words image representation introduced in
  Fisher kernels on visual vocabularies for image categorization
  F. Perronnin and C. Dance, CVPR 2007.

- FV in comparison to the BoW representation
  - Both FV and BoW are based on a visual vocabulary, with assignment of patches to visual words
  - FV based on Mixture of Gaussian clustering of patches, BoW based on k-means clustering
  - FV Extracts a larger image signature than the BoW representation for a given number of visual words

Fisher vector representation: Motivation

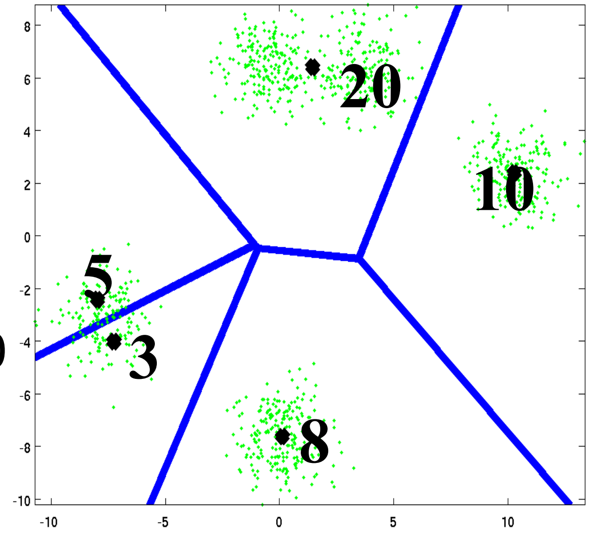
- Suppose we want to refine a given visual vocabulary

- Bag-of-word histogram stores # patches assigned to each word
  - Need more words to refine the representation
  - But this directly increases the computational cost
  - And leads to many empty bins: redundancy



Fisher vector representation: Motivation

- Feature vector quantization is computationally expensive
- To extract visual word histogram for a new image
  - Compute distance of each local descriptor to each k-means center
  - run-time O(NKD) : linear in
    - N: nr. of feature vectors ~ 10^4 per image
    - K: nr. of clusters ~ 10^3 for recognition
    - D: nr. of dimensions ~ 10^2 (SIFT)



- So in total in the order of 10^9 multiplications per image to obtain a histogram of size 1000
- Can this be done more efficiently ?!
  - Yes, extract more than just a visual word histogram

Fisher vector representation in a nutshell

- Instead, the Fisher Vector also records the mean and variance of the points per dimension in each cell
  - More information for same # visual words
  - Does not increase computational time significantly
  - Leads to high-dimensional feature vectors
- Even when the counts are the same,

  the position and variance of the points in the cell can vary

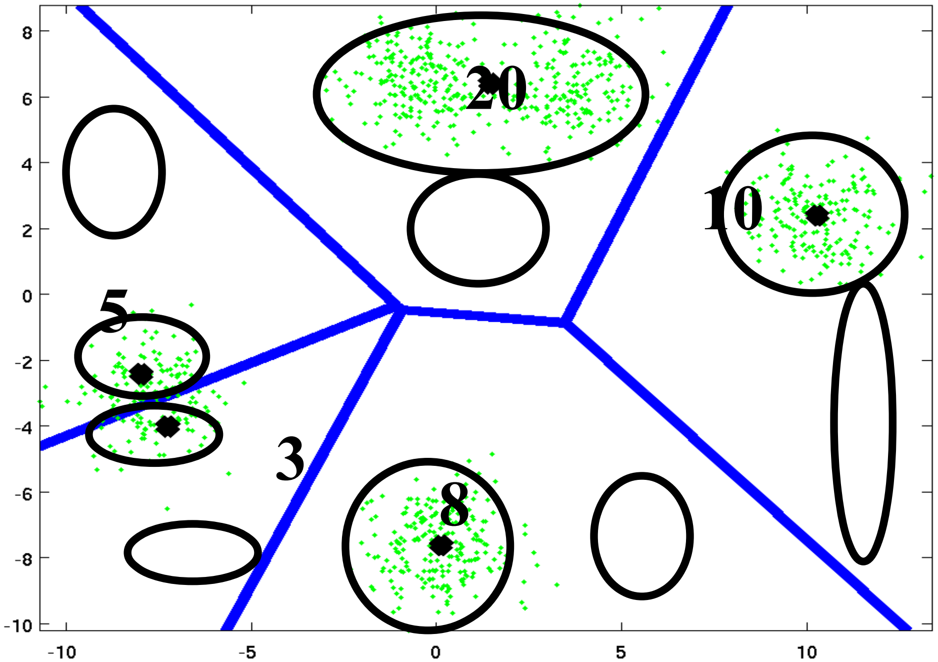


Image representation using Fisher kernels

- General idea of Fischer vector representation
  - Fit probabilistic model to data $p(X;\Theta)$
  - Represent data with derivative of data log-likelihood
    "How does the data want that the model changes?"

$$G(X,\Theta)=\frac{\partial \log p(x;\Theta)}{\partial \Theta}$$

Jaakkola & Haussler. "Exploiting generative models in discriminative classifiers", in Advances in Neural Information Processing Systems 11, 1999.

- Mixture of Gaussians to model the local (SIFT) descriptors $X=\{x_n\}_{n=1}^{N}$

$$L(X,\Theta)=\sum_n \log p(x_n)$$
$$p(x_n)=\sum_k \pi_k N(x_n;m_k,C_k)$$

  - Define mixing weights using the soft-max function ensures positiveness and sum to one constraint
  - Diagonal co-variance matrices

$$\pi_k=\frac{\exp \alpha_k}{\sum_{k'} \exp \alpha_{k'}}$$

Image representation using Fisher kernels

- Mixture of Gaussians to model the local (SIFT) descriptors

$$L(\Theta) = \sum_n \log p(x_n)$$
$$p(x_n) = \sum_k \pi_k N(x_n; m_k, C_k)$$

  - The parameters of the model are $\Theta = \{\alpha_k, m_k, C_k\}_{k=1}^{K}$
  - where we use diagonal covariance matrices

- Concatenate derivatives to obtain data representation

$$G(X,\Theta) = \left(\frac{\partial L}{\partial \alpha_1}, \ldots, \frac{\partial L}{\partial \alpha_K} \; , \; \frac{\partial L}{\partial m_1}, \ldots, \frac{\partial L}{\partial m_K} \; , \; \frac{\partial L}{\partial C_1^{-1}}, \ldots, \frac{\partial L}{\partial C_K^{-1}} \right)^T$$

informatics mathematics

Grenoble INP Ensimag

Image representation using Fisher kernels

- Data representation

$$G(X,\Theta)=\left(\frac{\partial L}{\partial \alpha_1},\ldots,\frac{\partial L}{\partial \alpha_K}\;,\;\frac{\partial L}{\partial m_1},\ldots,\frac{\partial L}{\partial m_K}\;,\;\frac{\partial L}{\partial C_1^{-1}},\ldots,\frac{\partial L}{\partial C_K^{-1}}\right)^T$$

- In total K(1+2D) dimensional representation,

  since for each visual word / Gaussian we have

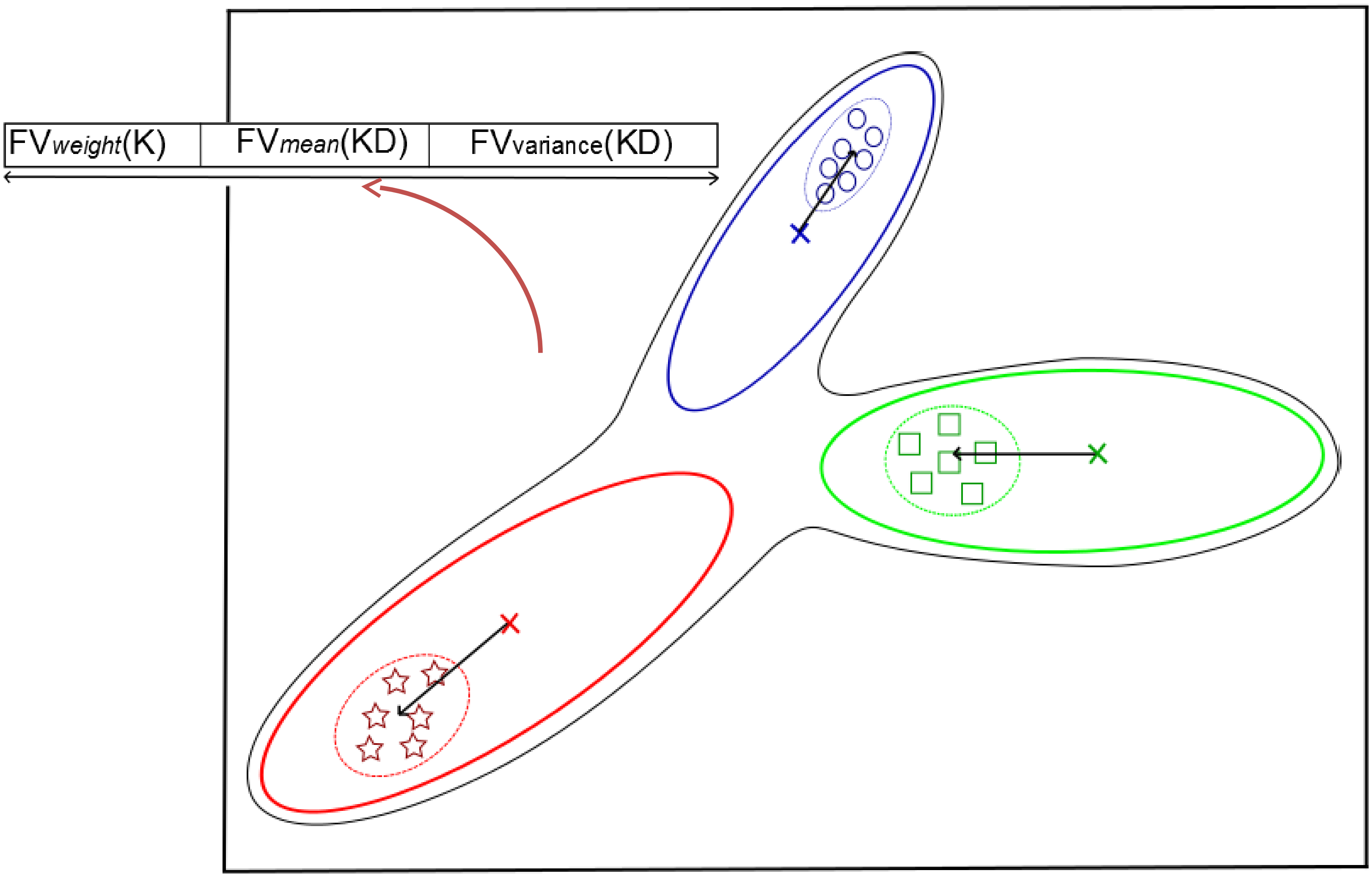
Count (1 dim) : $\frac{\partial L}{\partial \alpha_k}=\sum_n \left(q_{nk}-\pi_k\right)$ — More/less patches assigned to visual word than usual?

Mean (D dims) : $\frac{\partial L}{\partial m_k}=C_k^{-1}\sum_n q_{nk}\left(x_n-m_k\right)$ — Center of assigned data relative to cluster center

Variance (D dims) : $\frac{\partial L}{\partial C_k^{-1}}=\frac{1}{2}\sum_n q_{nk}\left(C_k-\left(x_n-m_k\right)^2\right)$ — Variance of assigned data relative to cluster variance

With the soft-assignments: $q_{nk}=p(k|x_n)=\frac{\pi_k p(x_n|k)}{p(x_n)}$

Illustrative example in 2d



New Data Points

Gradient with respect to mean

Function approximation view

- Suppose our local descriptors are 1 dimensional for simplicity
  - Vocabulary quantizes the real line
- Suppose we use a linear function, eg for image classification
  - BoW: locally constant function

$$f(x;w) = \sum_{k=1}^{K} x_k w_k$$

  - FV: locally constant + linear + quadratic function

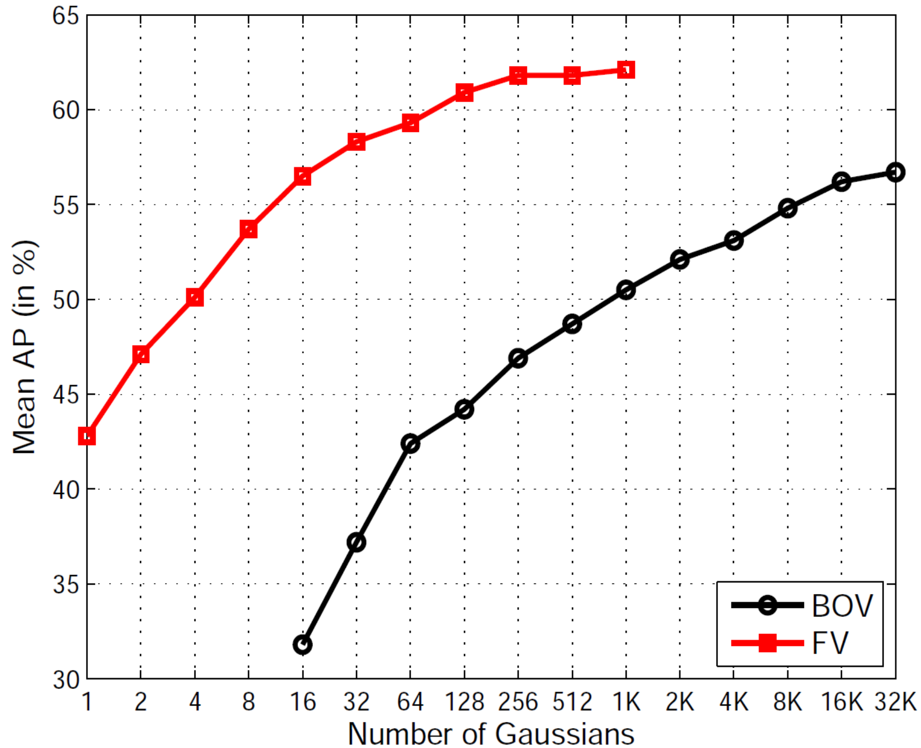
$$f(x;w) = \sum_{k=1}^{K} \left[\frac{\partial L}{\partial \alpha_k} \; \frac{\partial L}{\partial \mu_k} \; \frac{\partial L}{\partial C_k^{-1}} \right]^T w_k$$

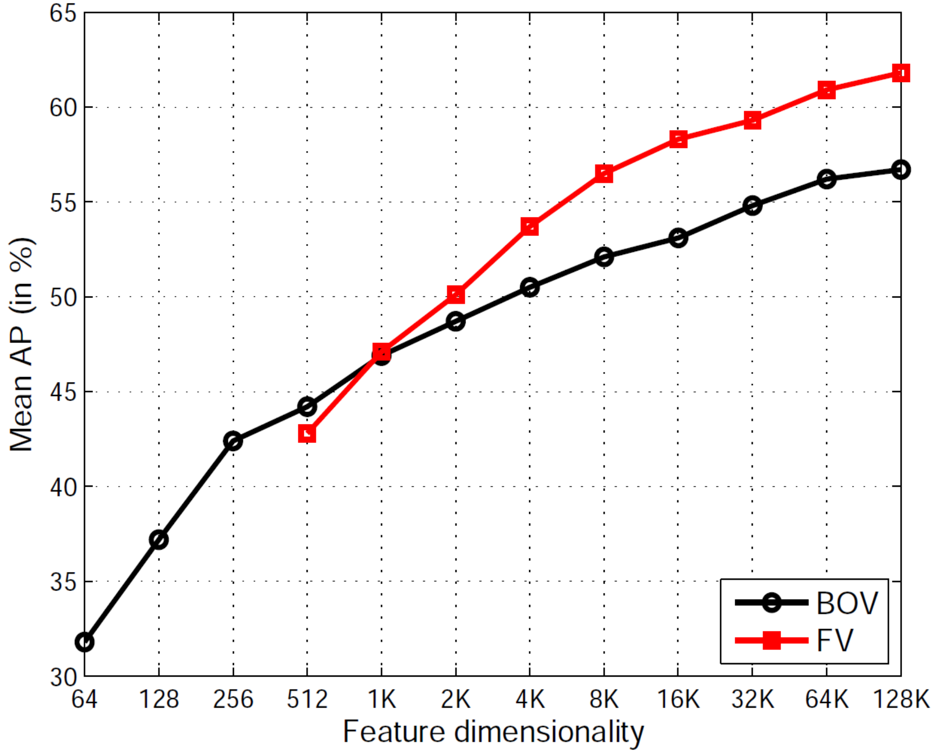
Images from categorization task PASCAL VOC

- Yearly evaluation from 2005 to 2012 for image classification

Fisher vectors: classification performance VOC'07

- Fisher vector representation yields better performance for a given number of Gaussians / visual words than Bag-of-words.
- For a fixed dimensionality Fisher vectors perform better, and are more efficient to compute





Bag-of-words vs. Fisher vector image representation

- Bag-of-words image representation
  - Off-line: fit k-means clustering to local descriptors
  - Represent image with histogram of visual word counts: K dimensions

- Fischer vector image representation
  - Off-line: fit MoG model to local descriptors
  - Represent image with gradient of log-likelihood: K(2D+1) dimensions

- Computational cost similar:
  - Both compare N descriptors to K visual words (centers / Gaussians)
- Memory usage: higher for fisher vectors
  - Fisher vector is a factor (2D+1) larger, e.g. a factor 257 for SIFTs !
    - For 1000 visual words the FV has 257,000 dimensions
  - However, because we store more information per visual word, we can generally obtain same or better performance with far less visual words

FV normalization

- Normalization with Fisher information matrix $F = E_{p(x)}[G(X,\Theta) G(X,\Theta)^T]$
  - Invariance w.r.t. re-parametrization, e.g. does not matter if we use standard dev., variance, or inverse-variance parameter

$$\tilde{G}(X,\Theta) = F^{-1/2} \left(\frac{\partial L}{\partial \alpha_1}, \dots, \frac{\partial L}{\partial \alpha_K} \;,\; \frac{\partial L}{\partial m_1}, \dots, \frac{\partial L}{\partial m_K} \;,\; \frac{\partial L}{\partial C_1^{-1}}, \dots, \frac{\partial L}{\partial C_K^{-1}} \right)^T$$

- Power normalization to reduce sparseness
  - Element-wise signed-power $\tilde{z} = sign(z) |z|^\rho$
  - Typically power set to 1/2, i.e. signed-square-root

- L2 normalization to make scales comparable
  - Eliminates effect of the number of patches
  - Increase FV magnitude for “typical” images with small gradient
  - Divide FV by its L2 norm

FV normalization, effect on performance

- Power normalization to reduce sparseness
- L2 normalization to make scales comparable
- Can also use Spatial Pyramids
  - compute FV per spatial cell, and concatenate
  - Here: 1x1, 2x2, 3x1 grid over image, 8 cells total

PN	ℓ_2	SP	SIFT	
No	No	No	49.6	
Yes	No	No	57.9	(+8.3)
No	Yes	No	54.2	(+4.6)
No	No	Yes	51.5	(+1.9)
Yes	Yes	No	59.6	(+10.0)
Yes	No	Yes	59.8	(+10.2)
No	Yes	Yes	57.3	(+7.7)
Yes	Yes	Yes	61.8	(+12.2)

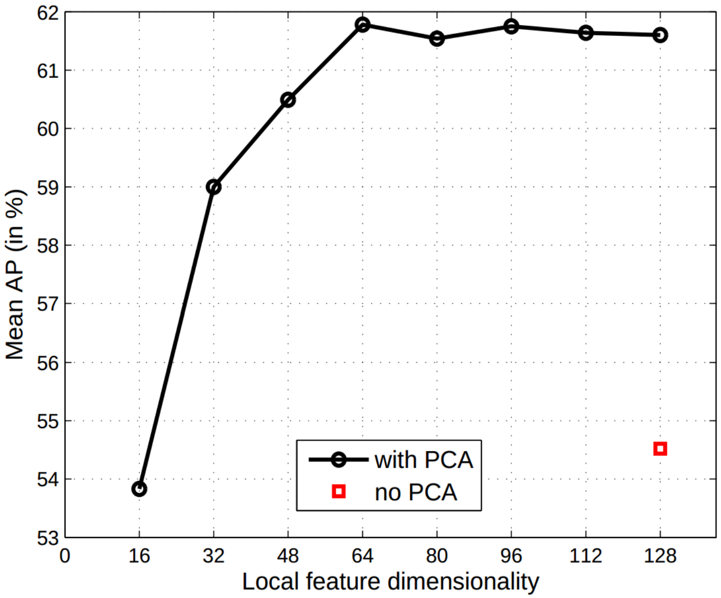
- Power + L2 normalization most important
- Spatial Pyramid also helps, but increases FV size by a factor 8

PCA projection of local features

- We used diagonal variances
  - Assumes dimensions are de-correlated
  - Not true for most local descriptors, like SIFT

- Perform PCA on the descriptors to de-correlate them
  - Possibly also reduce the dimension too

- Effect on image classification performance



Reading material

- A recent overview article on the Fisher Vector representation
  - Image Classification with the Fisher Vector: Theory and Practice
    Jorge Sanchez; Florent Perronnin; Thomas Mensink; Jakob Verbeek
    International Journal of Computer Vision, springer, 2013