# Efficient visual search of local features 

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## Visual search


change in viewing angle


## Matches



22 correct matches

## Image search system for large datasets



- Issues for very large databases
- to reduce the query time
- to reduce the storage requirements

Two strategies

1. Efficient approximate nearest neighbour search on local feature descriptors.
2. Quantize descriptors into a "visual vocabulary" and use efficient techniques from text retrieval. (Bag-of-words representation)

## Strategy 1: Efficient approximate NN search



1. Compute local features in each image independently
2. Describe each feature by a descriptor vector
3. Find nearest neighbour vectors between query and database
4. Rank matched images by number of (tentatively) corresponding regions
5. Verify top ranked images based on spatial consistency

## Finding nearest neighbour vectors

Establish correspondences between query image and images in the database by nearest neighbour matching on SIFT vectors


Solve following problem for all feature vectors, $\mathbf{x}_{j} \in \mathcal{R}^{128}$, in the query image:

$$
\forall j N N(j)=\arg \min _{i}\left\|\mathbf{x}_{i}-\mathbf{x}_{j}\right\|
$$

where, $\mathbf{x}_{i} \in \mathcal{R}^{128}$, are features from all the database images.

## Quick look at the complexity of the NN-search

N ... images
M ... regions per image (~1000)
D ... dimension of the descriptor ( $\sim 128$ )
Exhaustive linear search: O(M NMD)

## Example:

- Matching two images ( $\mathrm{N}=1$ ), each having 1000 SIFT descriptors Nearest neighbors search: 0.4 s ( 2 GHz CPU, implemenation in C)
- Memory footprint: 1000 * $128=128 \mathrm{kB} /$ image

| \# of images | CPU time | Memory req. |
| :---: | :---: | :---: |
| $\mathrm{N}=1,000$ | $\sim 7 \mathrm{~min}$ | ( $\sim 100 \mathrm{MB}$ ) |
| $N=10,000$ | $\sim 1 \mathrm{~h} 7 \mathrm{~min}$ | ( $\sim 1 \mathrm{~GB}$ ) |
| $\cdots$ |  |  |
| $\mathrm{N}=10^{7}$ | ~115 days | ( $\sim 1 \mathrm{~TB}$ ) |
| $\mathrm{N}=10^{10}$ | ~300 years | $(\sim 1 \mathrm{~PB})$ |

## Nearest-neighbor matching

Solve following problem for all feature vectors, $\mathbf{x}_{\mathrm{j}}$, in the query image:

$$
\forall j N N(j)=\arg \min _{i}\left\|\mathbf{x}_{i}-\mathbf{x}_{j}\right\|
$$

where $\mathbf{x}_{\mathbf{i}}$ are features in database images.

Nearest-neighbour matching is the major computational bottleneck

- Linear search performs dn operations for $n$ features in the database and dimensions
- No exact methods are faster than linear search for d>10
- Approximate methods can be much faster, but at the cost of missing some correct matches. Failure rate gets worse for large datasets.


## Approximate nearest neighbour search

- kd-trees (k dim. tree)
- Binary tree in which each node is a k-dimensional point
- Every split is associated with one dimension


kd-tree


## K-d tree

- K-d tree is a binary tree data structure for organizing a set of points
- Each internal node is associated with an axis aligned hyper-plane splitting its associated points into two sub-trees.
- Dimensions with high variance are chosen first.
- Position of the splitting hyper-plane is chosen as the mean/median of the projected points - balanced tree.



## K-d tree construction

Simple 2D example


## K-d tree query



## Large scale obiect/scene recognition



- Each image described by approximately 2000 descriptors
- 2 * $10^{9}$ descriptors to index for one million images!
- Database representation in RAM:
- Size of descriptors : 1 TB, search+memory intractable


## Bag-of-features [siviczZisserman'03]



- "visual words":

- 1 "word" (index) per local descriptor
- only images ids in inverted file
=> 8 GB fits!
Re-ranked $\qquad$ Geometric verification
$\qquad$ ranked image short-list
[Chum \& al. 2007]


## Indexing text with inverted files



Need to map feature descriptors to "visual words"

## Visual words: main idea

Map high-dimensional descriptors to tokens/words by quantizing the feature space


- Quantize via clustering, let cluster centers be the prototype "words"


## Visual words: main idea

Map high-dimensional descriptors to tokens/words by quantizing the feature space


- Determine which word to assign to each new image region by finding the closest cluster center.


## Visual words

-Example: each group of patches belongs to the same visual word


## K-means clustering

- Minimizing sum of squared Euclidean distances between points $x_{i}$ and their nearest cluster centers
- Algorithm:
- Randomly initialize K cluster centers
- Iterate until convergence:
- Assign each data point to the nearest center
- Recompute each cluster center as the mean of all points assigned to it
- Local minimum, solution dependent on initialization
- Initialization important, run several times, select best


## Inverted file index for images comprised of visual words


frame \#5

frame \#10


- Score each image by the number of common visual words (tentative correspondences)
- Dot product between bag-of-features
- Fast for sparse vectors !


## Inverted file index for images comprised of visual words

- Weighting with tf-idf score: weight visual words based on their frequency
-Tf: normalized term (word) ti frequency in a document dj

$$
t f_{i j}=n_{i j} / \sum_{k} n_{k j}
$$

-Idf: inverse document frequency, total number of documents divided by number of documents containing the term ti

$$
i d f_{i}=\log \frac{|D|}{\left|\left\{d: t_{i} \in d\right\}\right|}
$$

Tf-Idf:

$$
t f-i d f_{i j}=t f_{i j} \cdot i d f_{i}
$$

## Visual words

- Map descriptors to words by quantizing the feature space
- Quantize via k-means clustering to obtain visual words
- Assign descriptor to closest visual word
- Bag-of-features as approximate nearest neighbor search

Bag-of-features matching function $f_{q}(x, y)=\delta_{q(x), q(y)}$
where $\mathrm{q}(\mathrm{x})$ is a quantizer, i.e., assignment to visual word and $\delta_{a, b}$ is the Kronecker operator ( $\delta_{a, b}=1$ iff $a=b$ )

## Approximate nearest neighbor search evaluation

-ANN algorithms usually returns a short-list of nearest neighbors

- this short-list is supposed to contain the NN with high probability
- exact search may be performed to re-order this short-list
-Proposed quality evaluation of ANN search: trade-off between
- Accuracy: NN recall = probability that the NN is in this list


## against

- Ambiguity removal = proportion of vectors in the short-list
- the lower this proportion, the more information we have about the vector
- the lower this proportion, the lower the complexity if we perform exact search on the short-list
-ANN search algorithms usually have some parameters to handle this trade-off


## ANN evaluation of bag-of-features


-ANN algorithms returns a list of potential neighbors
-Accuracy: NN recall = probability that the NN is in this list
-Ambiguity removal:
= proportion of vectors in the short-list
-In BOF, this trade-off is managed by the number of clusters $k$

## Vocabulary size

- The intrinsic matching scheme performed by BOF is weak
- for a "small" visual dictionary: too many false matches
- for a "large" visual dictionary: complexity, true matches are missed
- No good trade-off between "small" and "large" !
- either the Voronoi cells are too big
- or these cells can't absorb the descriptor noise
$\rightarrow$ intrinsic approximate nearest neighbor search of BOF is not sufficient


## 20K visual word: false matches



## 200K visual word: good matches missed



## Hamming Embedding [Jegou etal. Eccvos]



Representation of a descriptor $x$

- Vector-quantized to $q(x)$ as in standard BOF
+ short binary vector $b(x)$ for an additional localization in the Voronoi cell
Two descriptors x and y match iif
$f_{\mathrm{HE}}(x, y)=\left\{\begin{array}{ll}(\mathrm{tf}-\mathrm{idf}(q(x)))^{2} & \begin{array}{c}\text { if } q(x)=q(y) \\ \text { and } h(b(x), b(y)) \leq h_{t} \\ 0\end{array} \\ \text { otherwise }\end{array} \quad\right.$ where $\mathrm{h}(a, b)$ Hamming distance


## Hamming Embedding

-Nearest neighbors for Hamming distance $\approx$ those for Euclidean distance
$\rightarrow$ a metric in the embedded space reduces dimensionality curse effects
-Efficiency

- Hamming distance = very few operations
- Fewer random memory accesses: $3 \times$ faster that BOF with same dictionary size!


## Hamming Embedding

-Off-line (given a quantizer)

- draw an orthogonal projection matrix $P$ of size $d_{b} \times d$
$\rightarrow$ this defines $\mathrm{d}_{\mathrm{b}}$ random projection directions
- for each Voronoi cell and projection direction, compute the median value for a learning set
-On-line: compute the binary signature $b(x)$ of a given descriptor
- project $x$ onto the projection directions as $z(x)=\left(z_{1}, \ldots z_{d b}\right)$
$-b_{i}(x)=1$ if $z_{i}(x)$ is above the learned median value, otherwise 0


## ANN evaluation of Hamming Embedding


compared to BOW: at least 10 times less points in the short-list for the same level of accuracy

Hamming Embedding provides a much better trade-off between recall and ambiguity removal

## Matching points - 20k word vocabulary

$$
201 \text { matches } \quad 240 \text { matches }
$$



Many matches with the non-corresponding image!

## Matching points - 200k word vocabulary

69 matches 35 matches


Still many matches with the non-corresponding one

## Matching points - 20k word vocabulary + HE




10x more matches with the corresponding image!

## Bag-of-features [siviczzisserman'03]


sparse frequency vector


- "visual words":

- 1 "word" (index) per local descriptor
- only images ids in inverted file
=> 8 GB fits!



## Geometric verification

Use the position and shape of the underlying features to improve retrieval quality


Both images have many matches - which is correct?

## Geometric verification

We can measure spatial consistency between the query and each result to improve retrieval quality


Many spatially consistent matches - correct result


Few spatially consistent matches - incorrect result

## Geometric verification

## Gives localization of the object



## Geometric verification

- Remove outliers, matches contain a high number of incorrect ones
- Estimate geometric transformation
- Robust strategies
- RANSAC
- Hough transform


## Example: estimating 2D affine transformation

- Simple fitting procedure (linear least squares)
- Approximates viewpoint changes for roughly planar objects and roughly orthographic cameras
- Can be used to initialize fitting for more complex models


Matches consistent with an affine transformation

## Fitting an affine transformation

Assume we know the correspondences, how do we get the transformation?


Fitting an affine transformation

$$
\left[\begin{array}{cccccc} 
& \mathrm{L} & & & & -\begin{array}{c}
m_{1} \\
m_{2} \\
x_{2}
\end{array} \\
y_{i} & 0 & 0 & 1 & 0 \\
m_{3} & 0 & x_{i} & y_{i} & 0 & 1 \\
m_{3} \\
m_{4} \\
t_{1} \\
t_{2}
\end{array}\right]=\left[\begin{array}{l}
\mathrm{L} \\
x_{i}^{\prime} \\
y_{i}^{\prime} \\
\mathrm{L}
\end{array}\right]
$$

Linear system with six unknowns
Each match gives us two linearly independent
equations: need at least three to solve for the transformation parameters

