

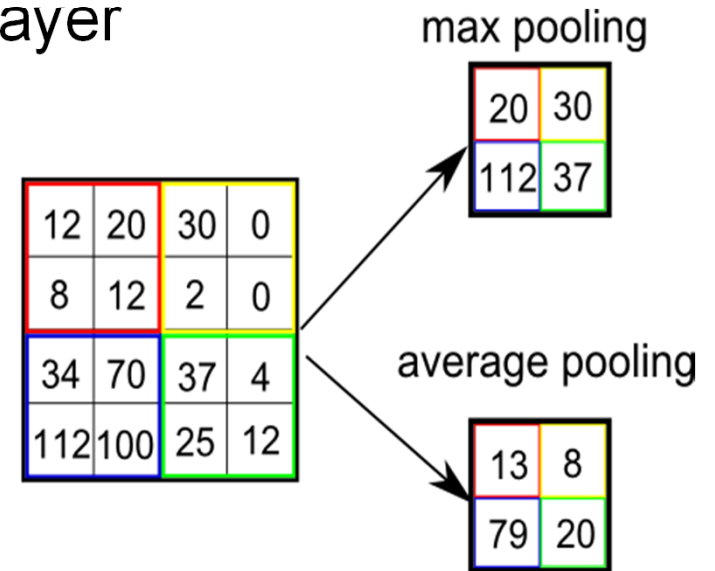
# Spatial Transformers in Feed-Forward Networks

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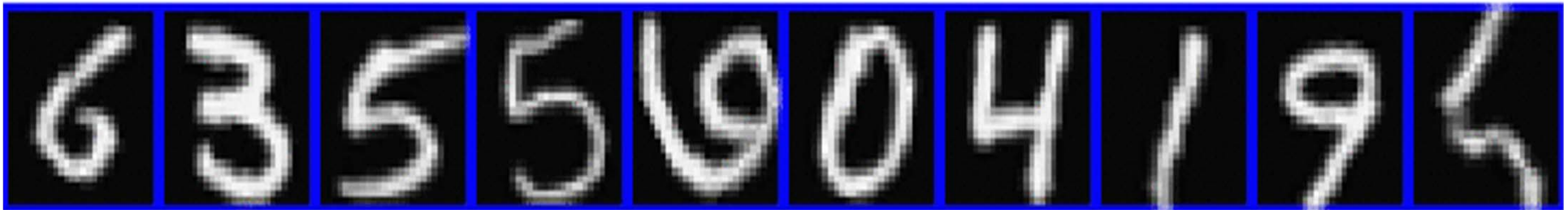
# ConvNets

- Interleaving convolutional layers with max-pooling layers allows translation invariance.
  - Pooling is simplistic.
  - Only small invariances per pooling layer
  - Limited spatial transformation
  - Pools across entire image
  - + Exceptionally effective
- Can we do better?



# Motivation 1: transformations of input data

Rotated MNIST (+/- 90°)



## Motivation 2: attention

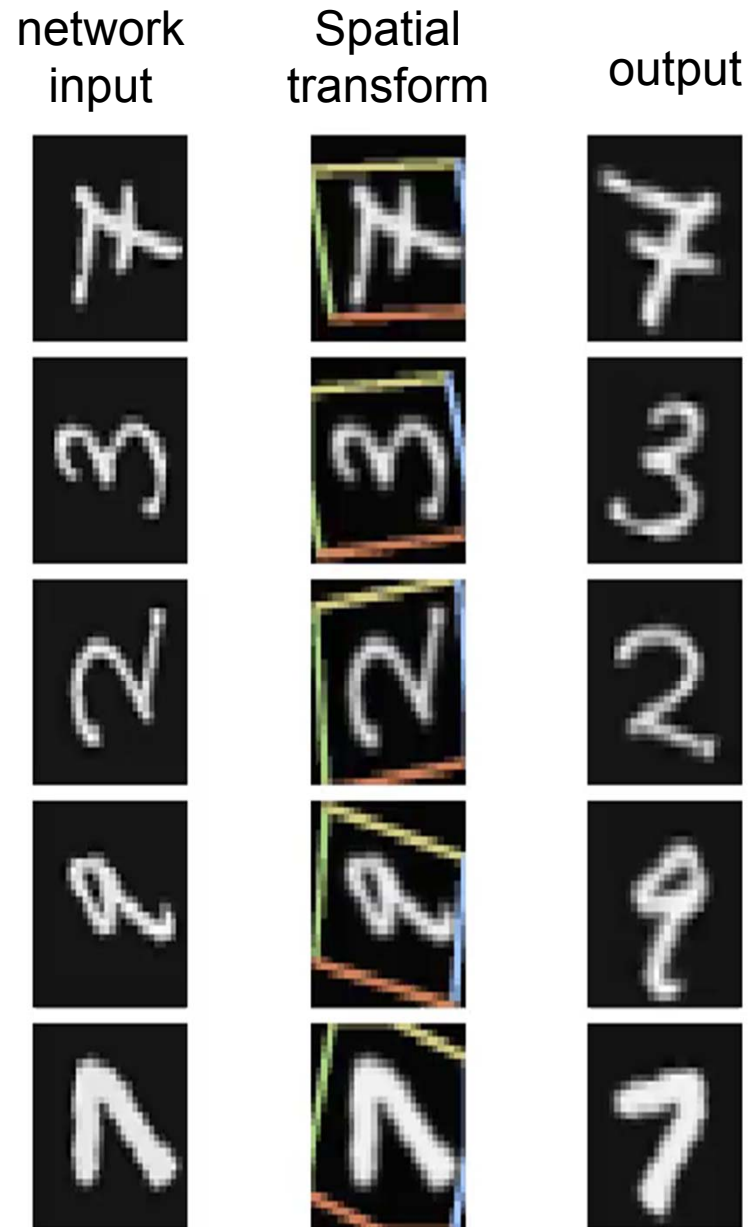


# Conditional Spatial Warping

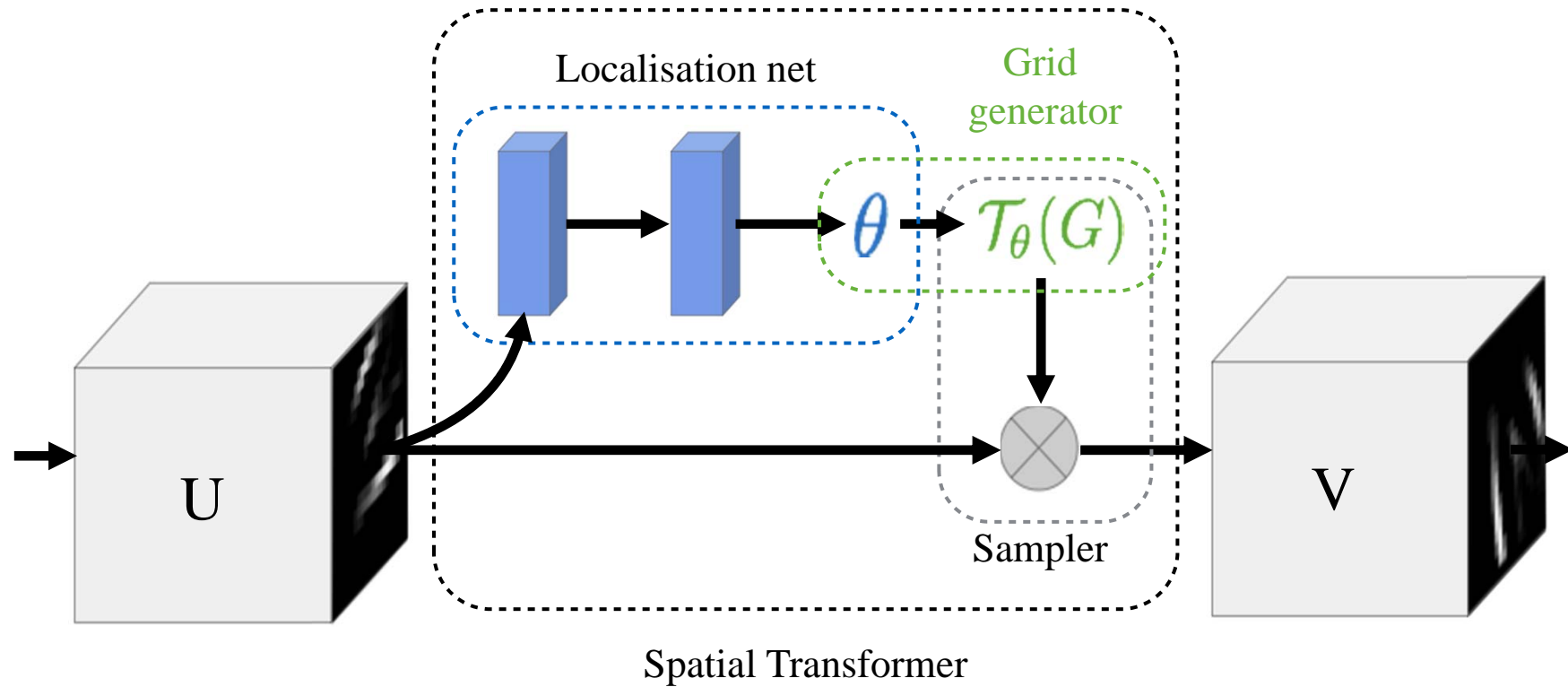
- Conditional on input featuremap, spatially warp image.
  - + Transforms data to a space expected by subsequent layers
  - + Intelligently select features of interest (attention)
  - + Invariant to more generic warping



# Conditional Spatial Warping



A differentiable module for spatially transforming data, conditional on the data itself

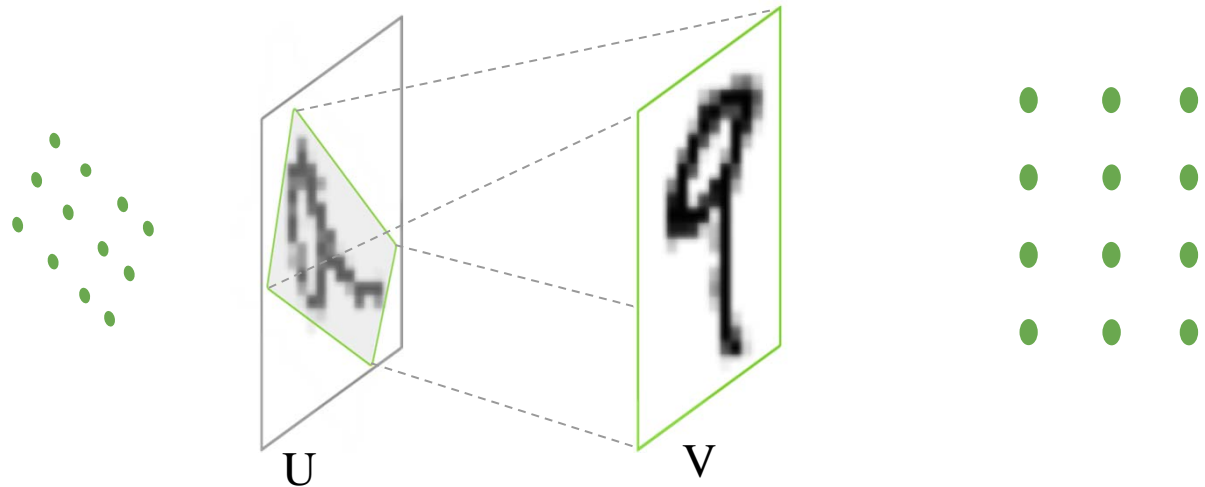


# Sampling Grid

Warp regular grid by an affine transformation

Can parameterise, e.g. affine transformation

$$\begin{pmatrix} x_i^s \\ y_i^s \end{pmatrix} = \mathcal{T}_\theta(G_i) = \mathbf{A}_\theta \begin{pmatrix} x_i^t \\ y_i^t \\ 1 \end{pmatrix} = \begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} \\ \theta_{21} & \theta_{22} & \theta_{23} \end{bmatrix} \begin{pmatrix} x_i^t \\ y_i^t \\ 1 \end{pmatrix}$$



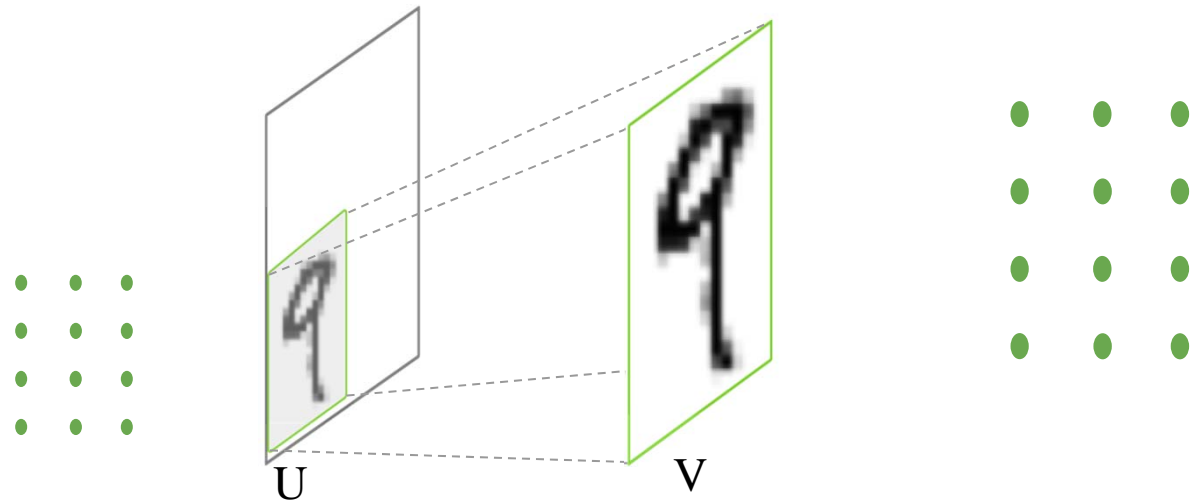


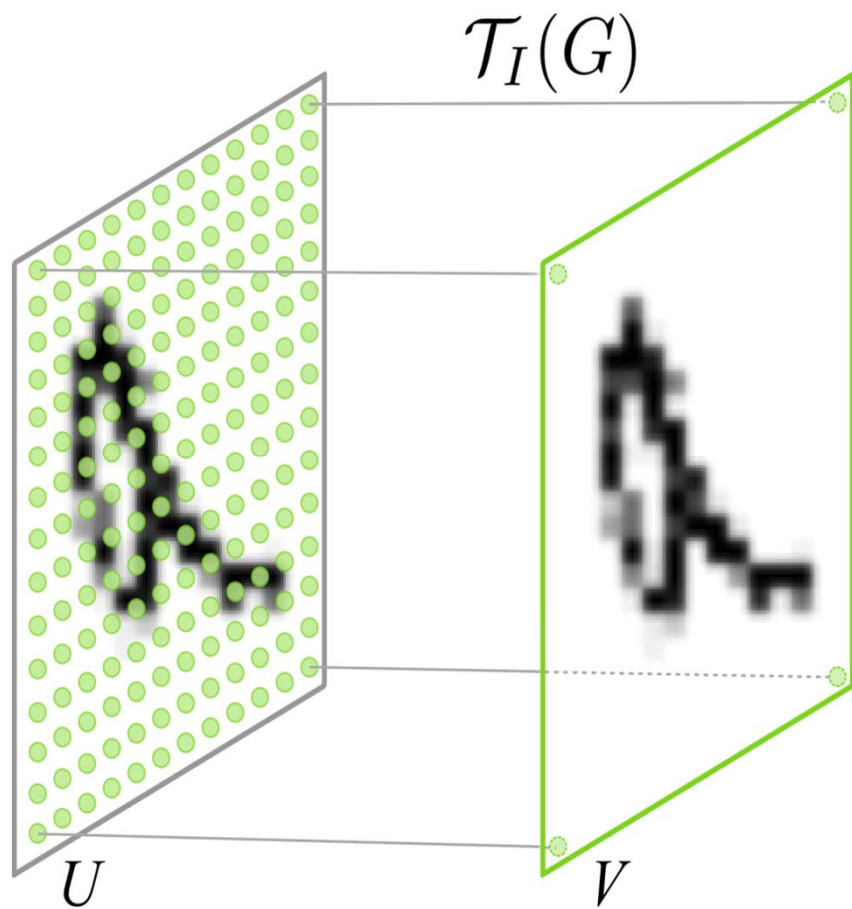
# Sampling Grid

Warp regular grid by an affine transformation

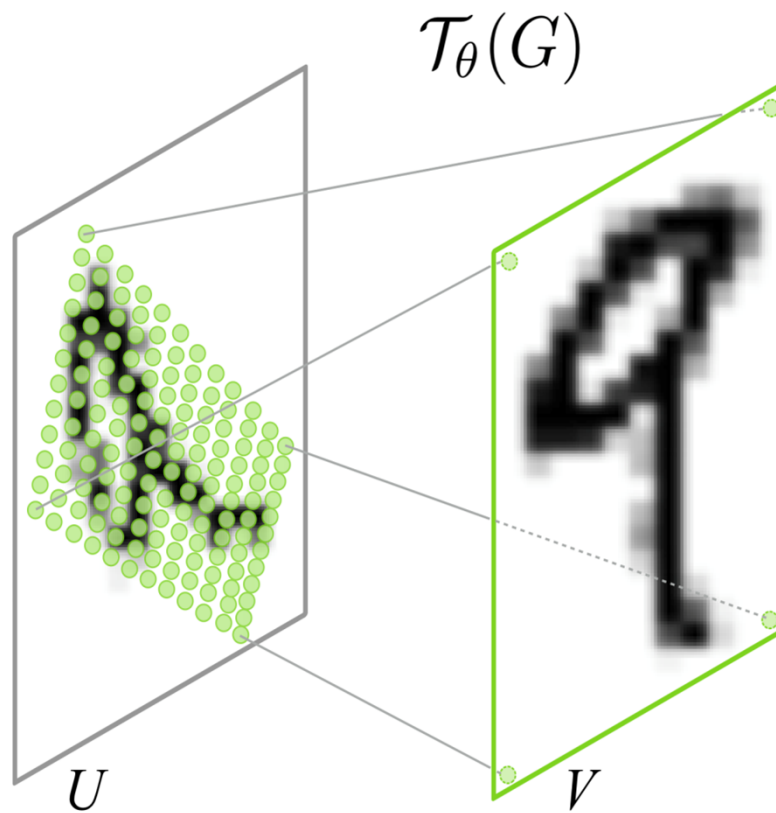
Can parameterise attention

$$\begin{pmatrix} x_i^s \\ y_i^s \end{pmatrix} = \mathcal{T}_\theta(G_i) = \mathbf{A}_\theta \begin{pmatrix} x_i^t \\ y_i^t \\ 1 \end{pmatrix} = \begin{bmatrix} s & 0 & t_x \\ 0 & s & t_y \end{bmatrix} \begin{pmatrix} x_i^t \\ y_i^t \\ 1 \end{pmatrix}$$





Identity  
transformation



affine  
transformation

# Sampler

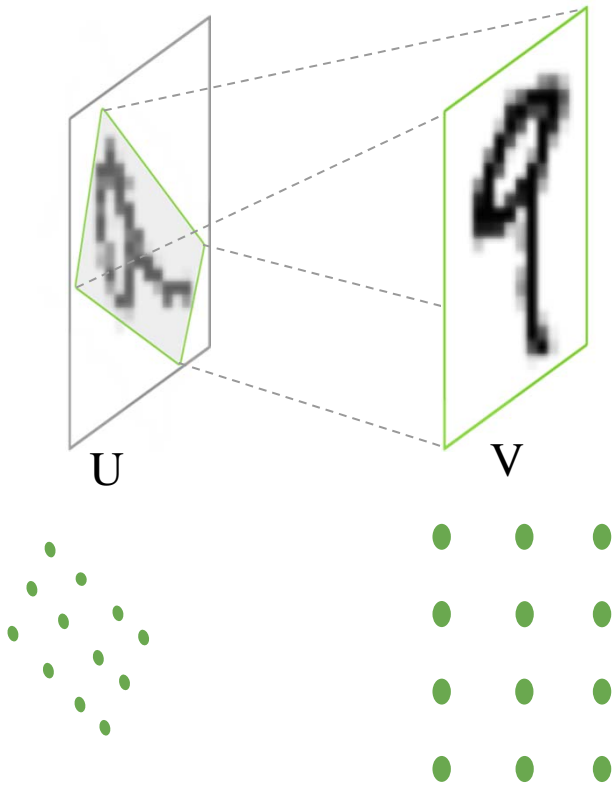
Sample input featuremap U to produce output feature map V (i.e. texture mapping)

e.g. for bilinear interpolation:

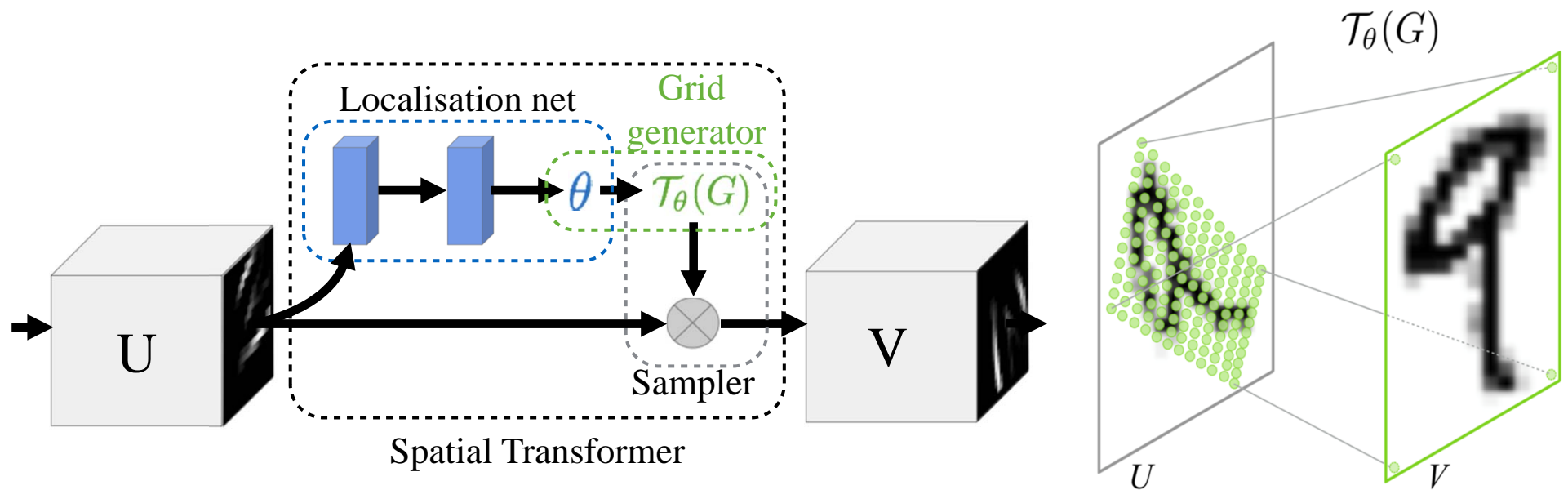
$$V_i^c = \sum_n^H \sum_m^W U_{nm}^c \max(0, 1 - |x_i^s - m|) \max(0, 1 - |y_i^s - n|)$$

and gradients are defined to allow backprop, eg:

$$\frac{\partial V_i^c}{\partial U_{nm}^c} = \sum_n^H \sum_m^W \max(0, 1 - |x_i^s - m|) \max(0, 1 - |y_i^s - n|)$$

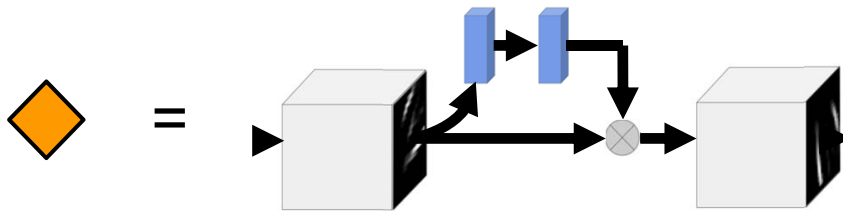


A differentiable module for spatially transforming data, conditional on the data itself



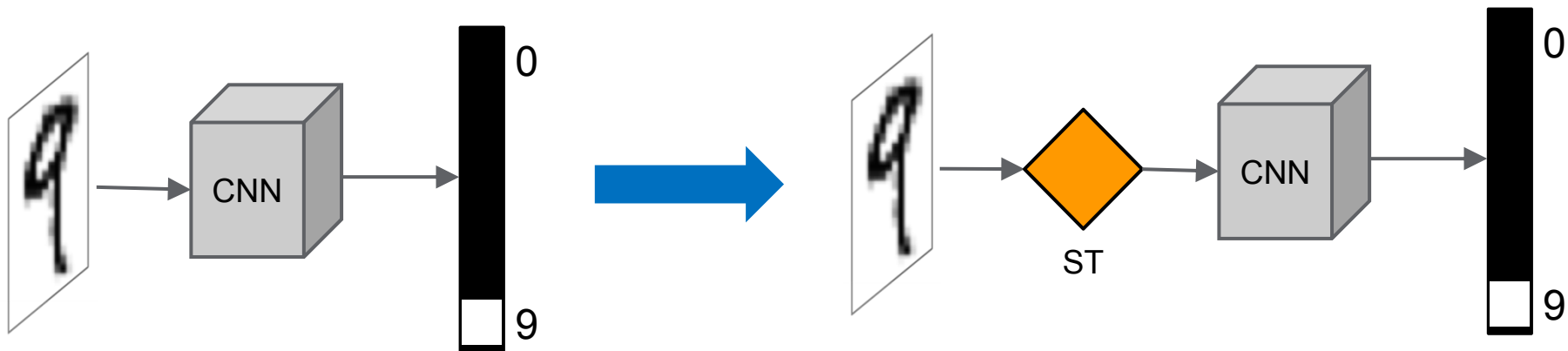
# Spatial Transformer Networks

- Spatial Transformers is differentiable, and so can be inserted at any point in a feed forward network and trained by back propagation



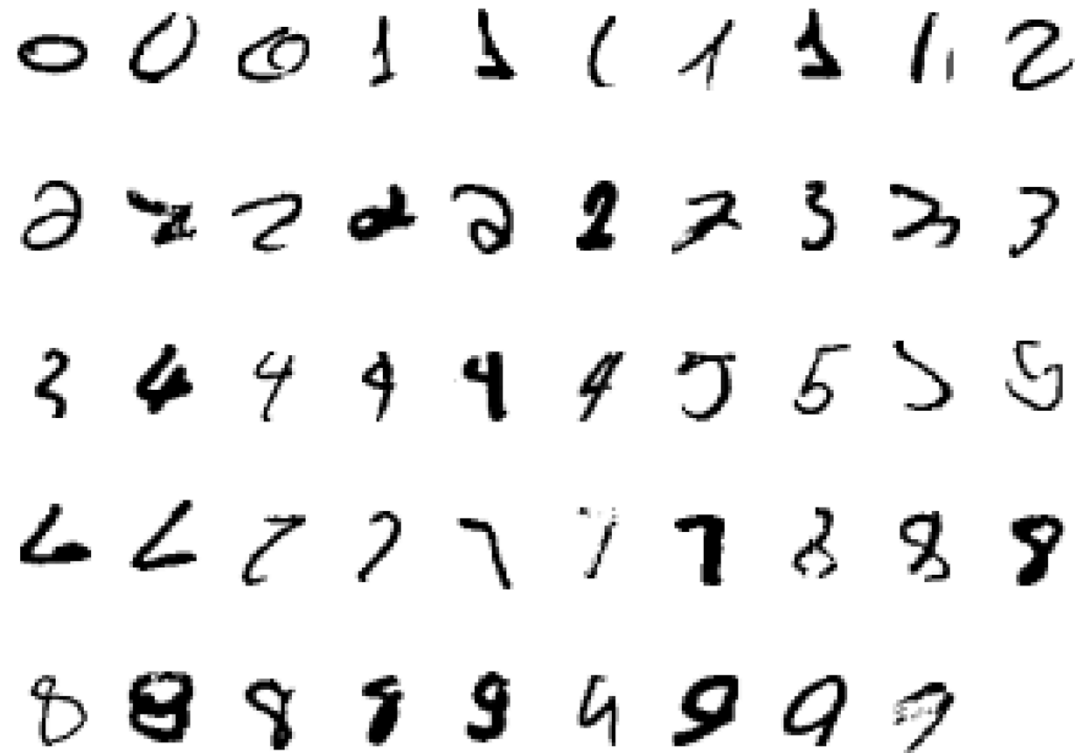
## Example:

- digit classification, loss: cross-entropy for 10 way classification



# MNIST Digit Classification

Training data: 6000 examples of each digit

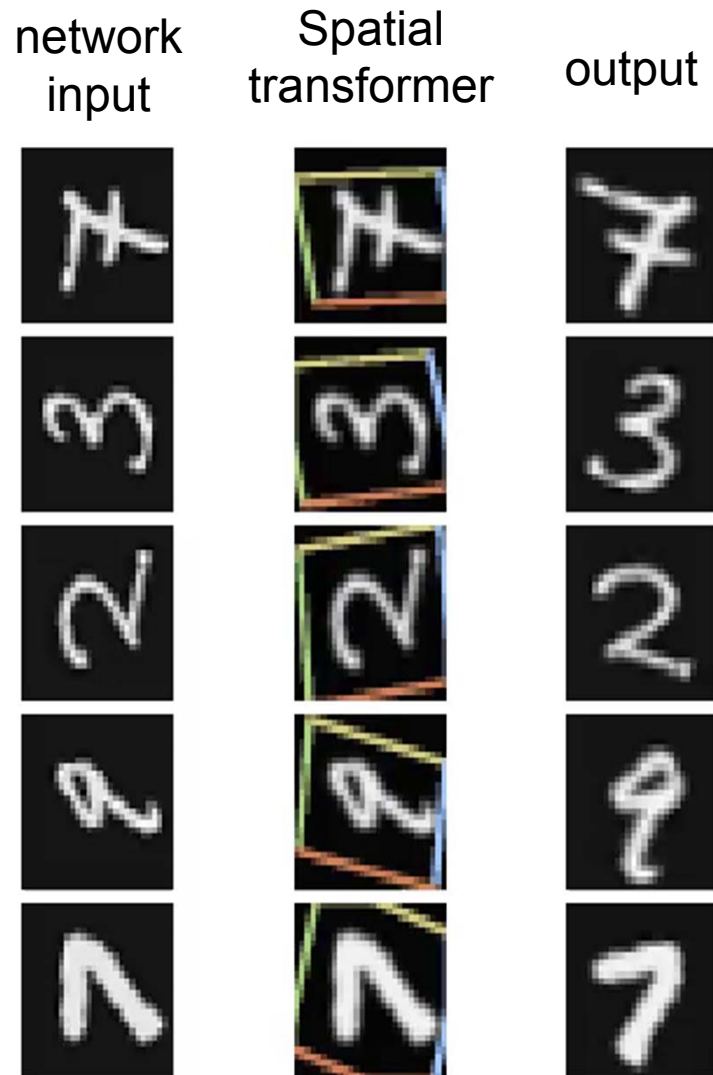


Testing data: 10k images

Can achieve testing error of 0.23%

# Task: classify MNIST digits

- Training and test randomly rotated by (+/- 90°)
- Fully connected network with affine ST on input



Performance:

- FCN 2.1
- CNN 1.2
- ST-FCN 1.2
- ST-CNN 0.7

# Generalizations 1: transformations

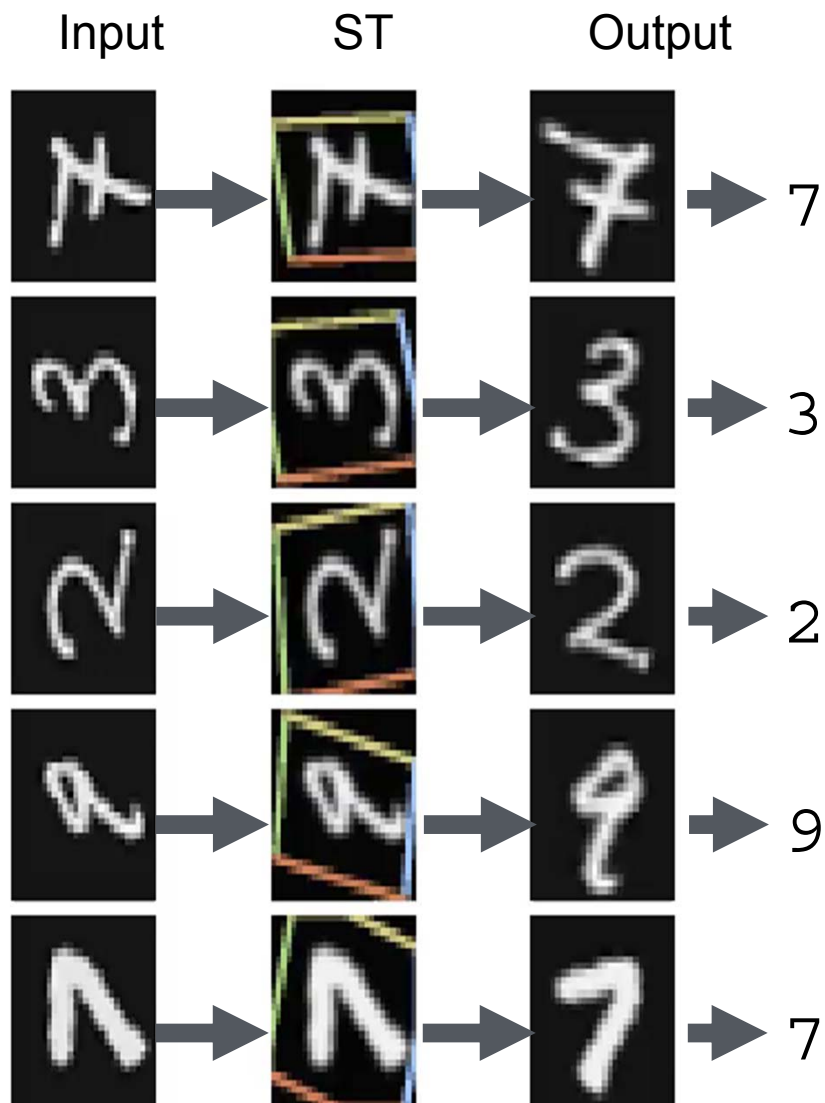
- Affine transformation – 6 parameters
- Projective transformation – 8 parameters
- Thin plate spline transformation
- Etc

Any transformation where parameters can be regressed

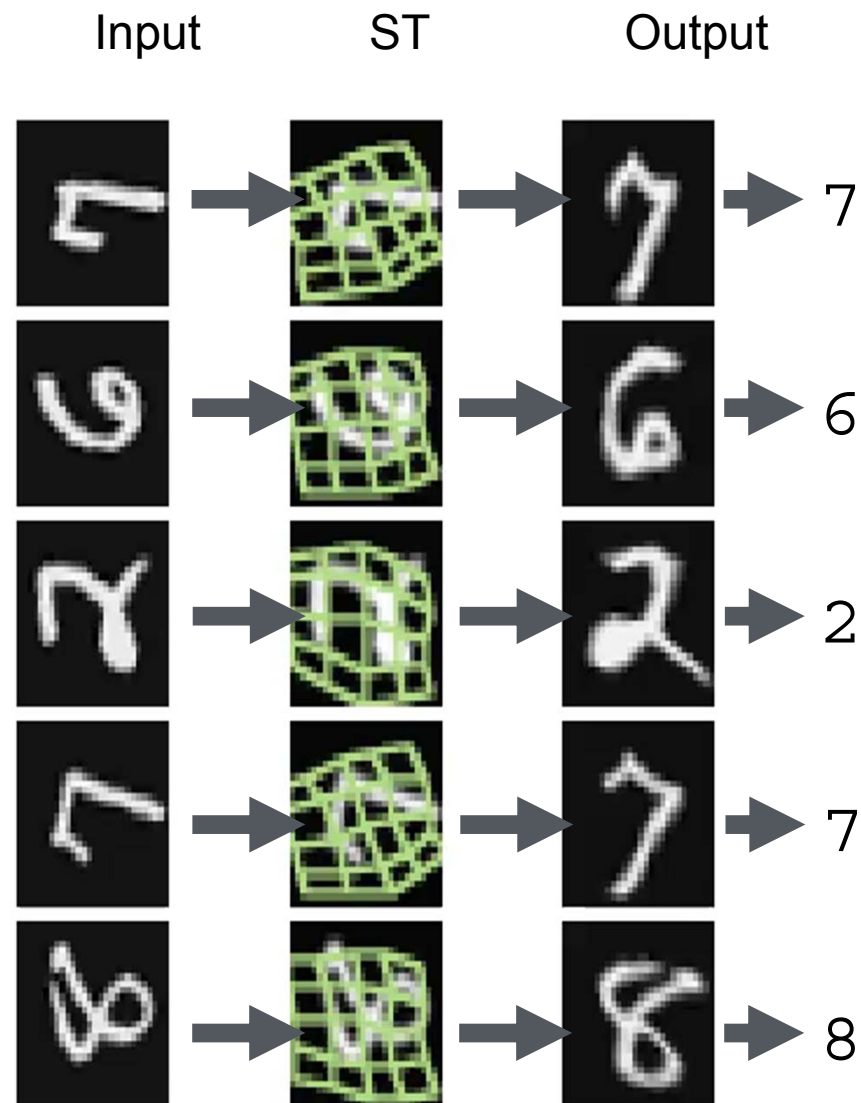


# Rotated MNIST

## ST-FCN Affine

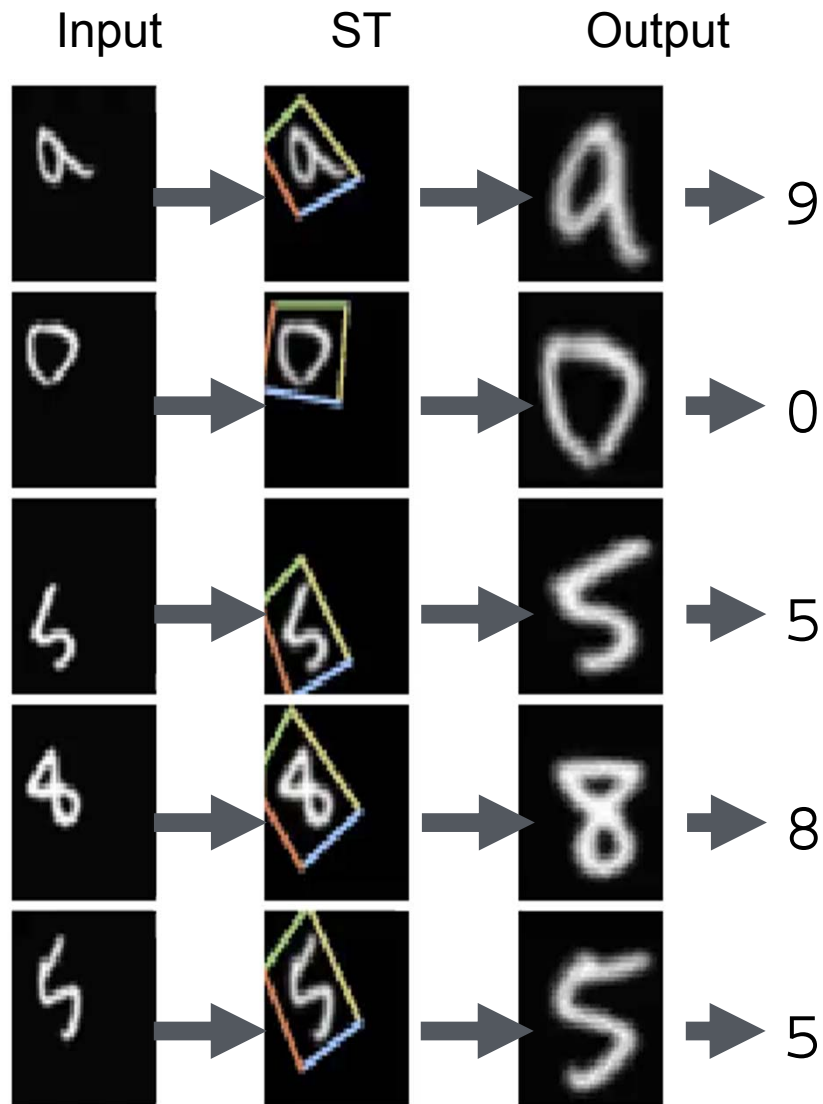


## ST-FCN Thin Plate Spline

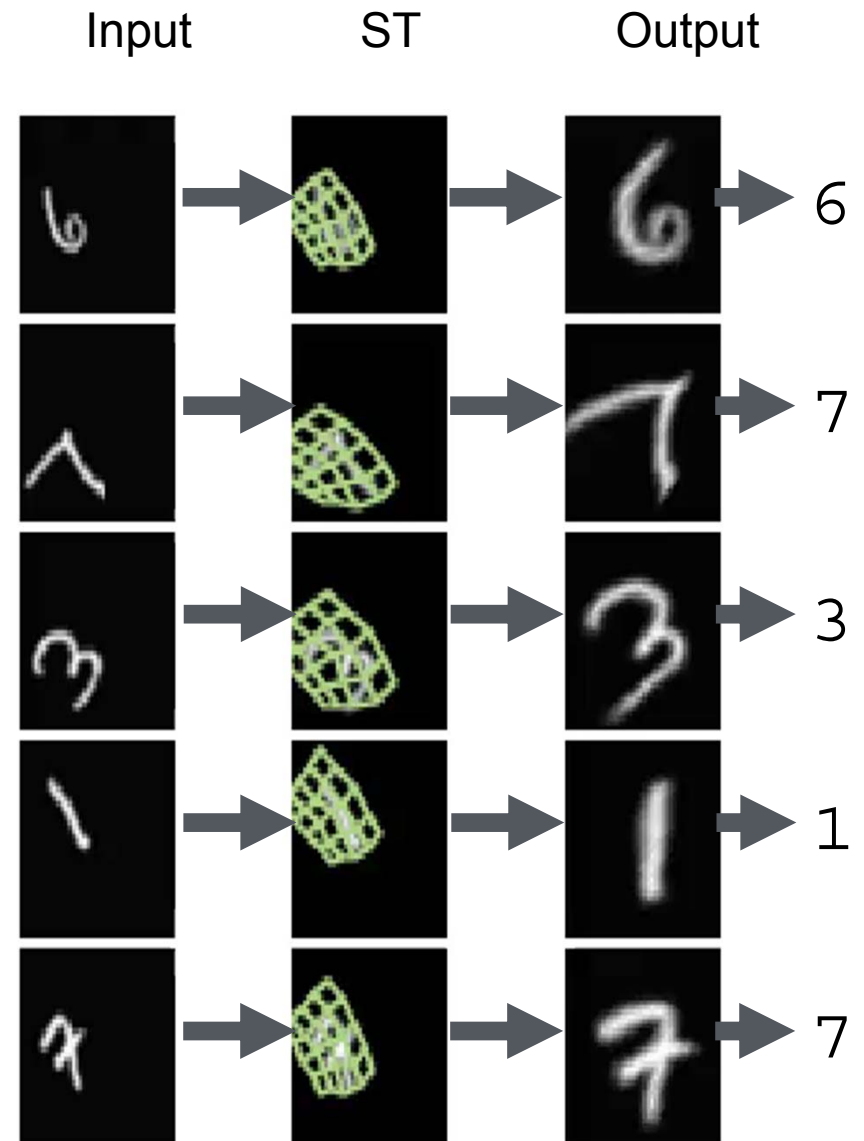


# Rotated, Translated & Scaled MNIST

## ST-FCN Projective

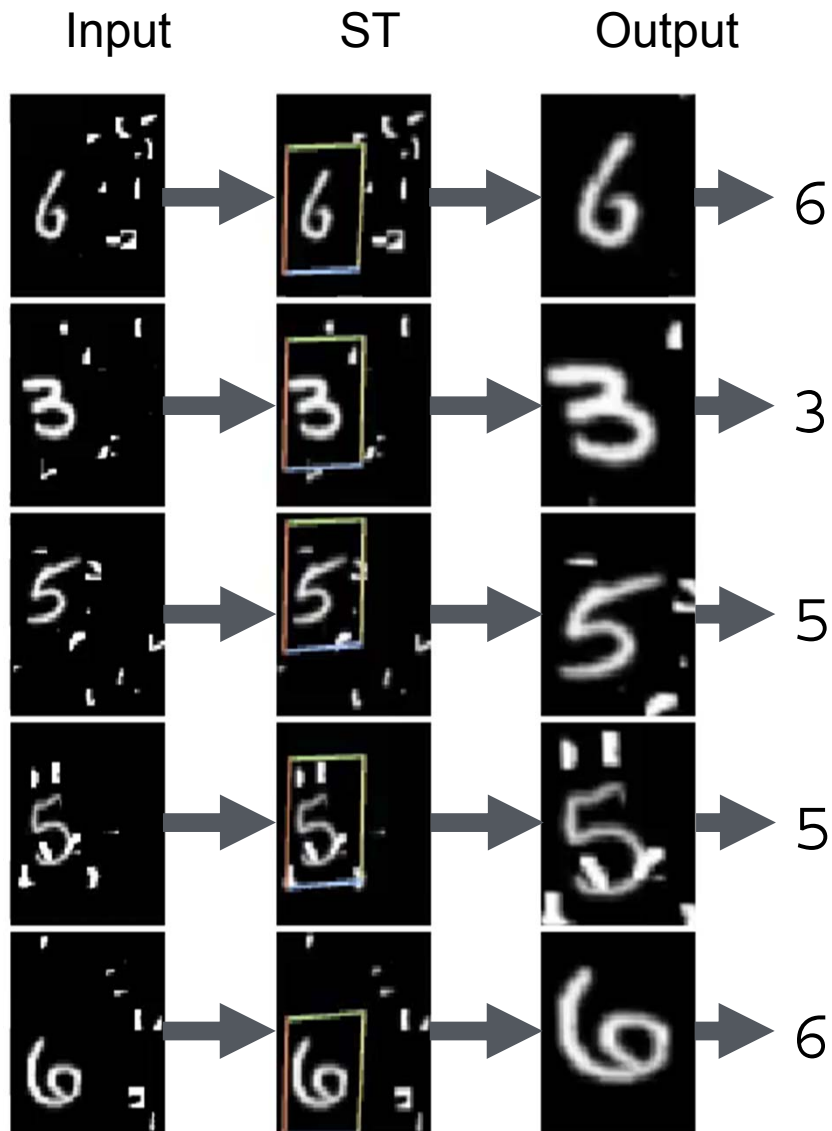


## ST-FCN Thin Plate Spline

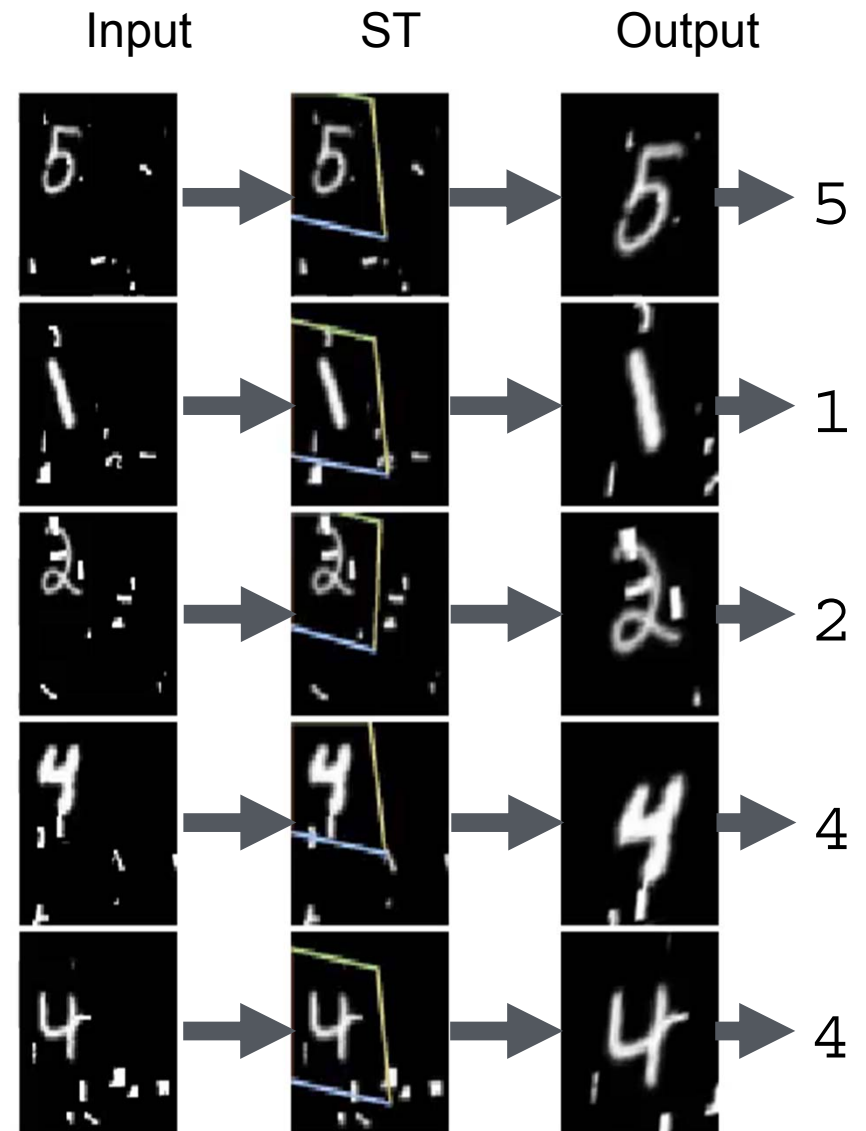


# Translated Cluttered MNIST

## ST-FCN Affine

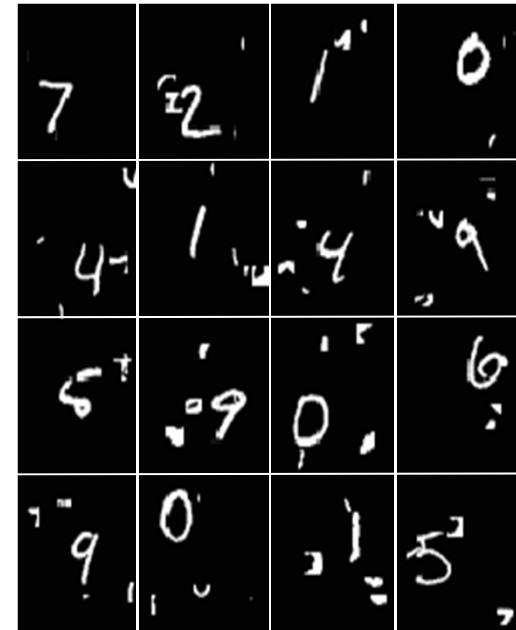


## ST-CNN Affine



# Results on performance

Model		MNIST Distortion			
		R	RTS	P	E
FCN		2.1	5.2	3.1	3.2
CNN		1.2	0.8	1.5	1.4
ST-FCN	Aff	1.2	0.8	1.5	2.7
	Proj	1.3	0.9	1.4	2.6
	TPS	1.1	0.8	1.4	2.4
ST-CNN	Aff	0.7	0.5	0.8	1.2
	Proj	0.8	0.6	0.8	1.3
	TPS	0.7	0.5	0.8	1.1



R: rotation

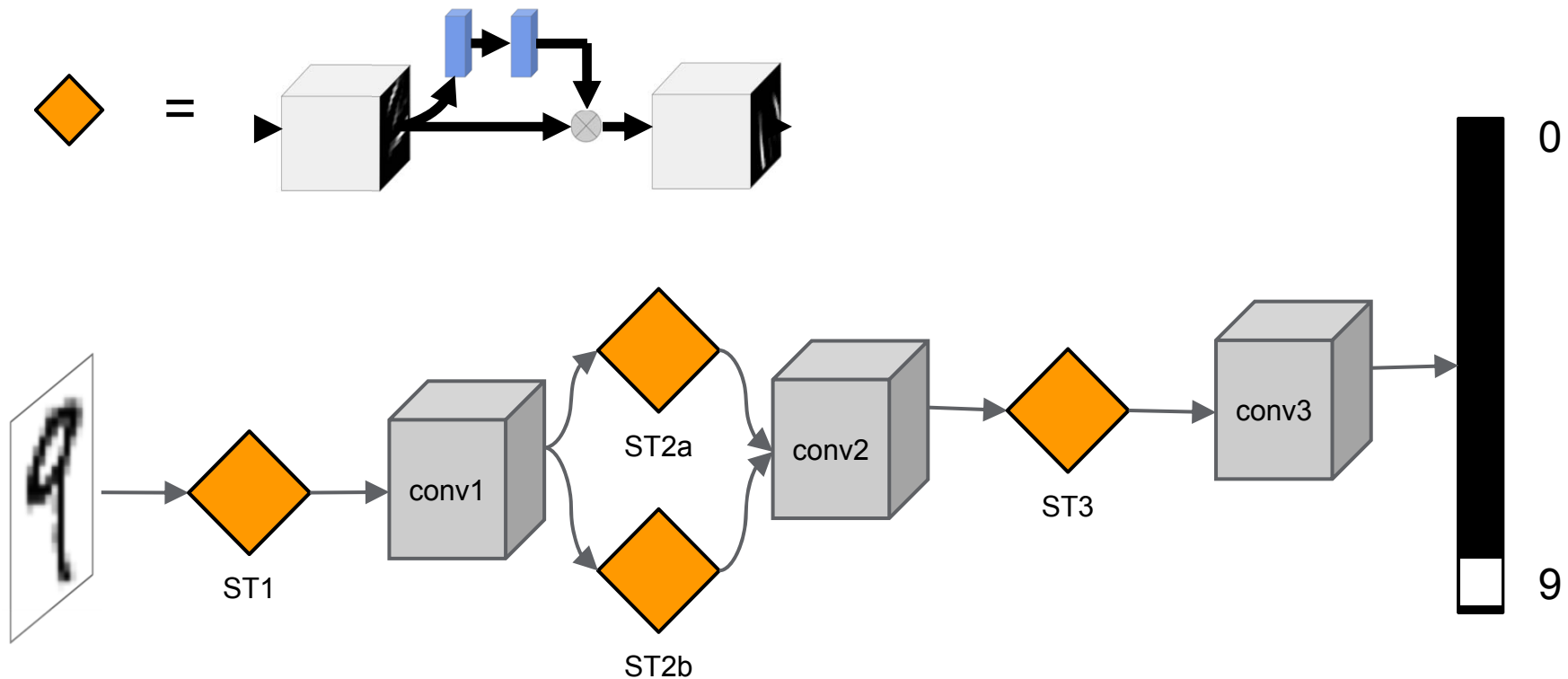
RTS: rotation, translation, scale

P: projective

E: elastic

# Generalization 2: Multiple spatial transformers

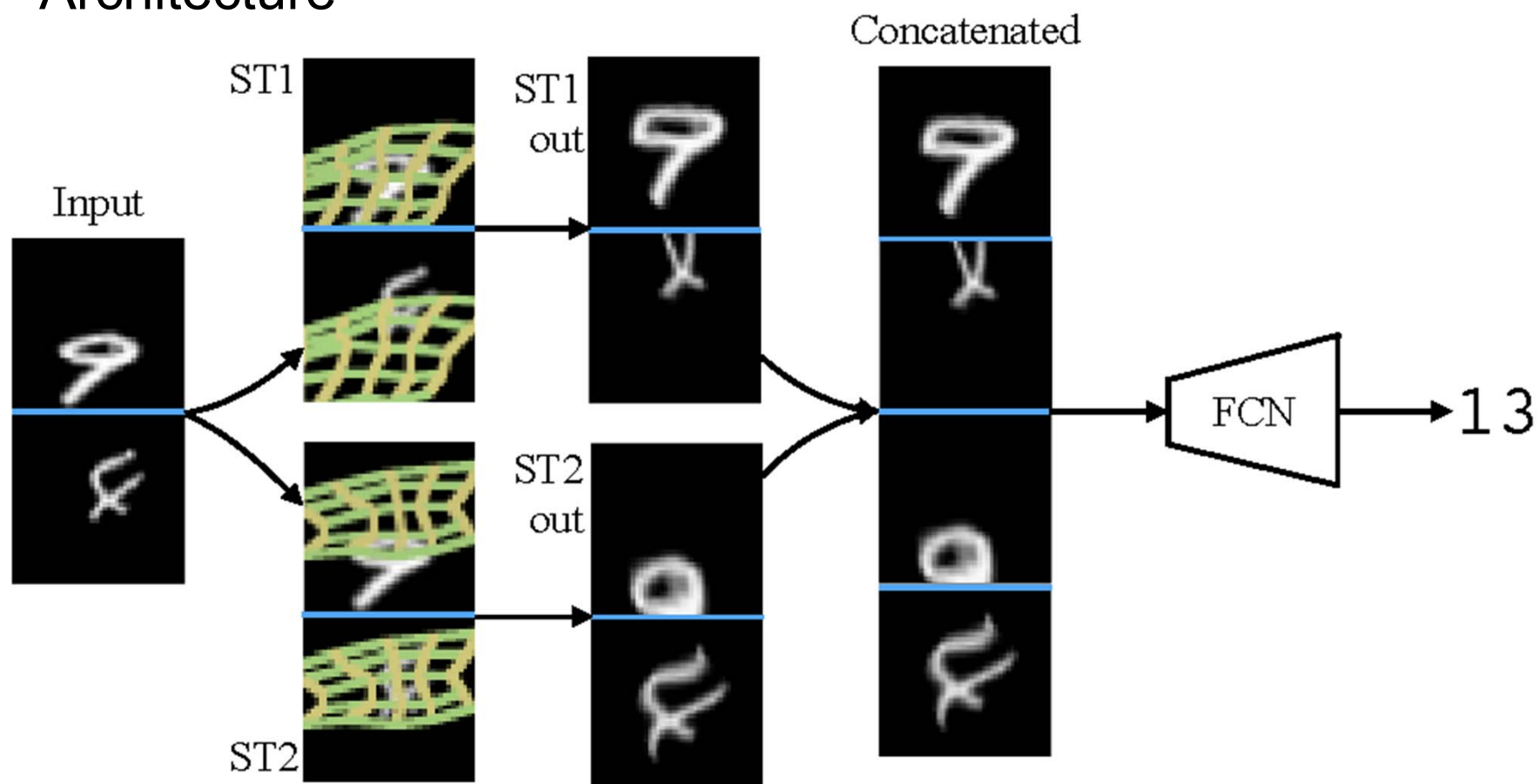
- Spatial Transformers can be inserted before/after conv layers, before/after max-pooling
- Can also have multiple Spatial Transformers at the same level



# Task: Add digits in two images

MNIST digits under rotation, translation and scale

## Architecture

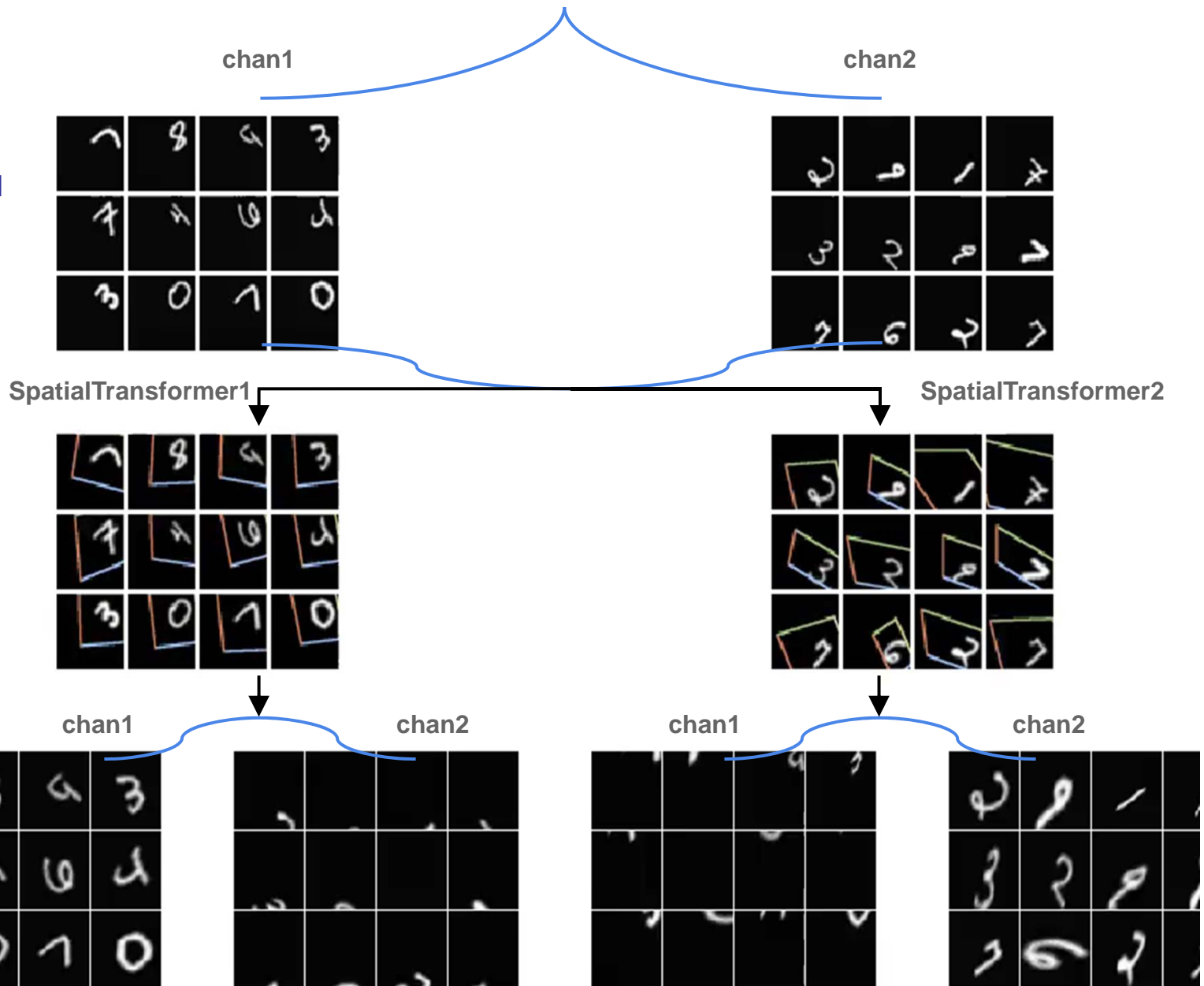


# Task: Add digits in two images

input (2 channels)

MNIST 2-channel  
addition

Add up the digits.  
One per channel.  
Random per-channel  
rotation, scale and  
translation.



SpatialTransformer1  
**automatically** specialises  
to rectify channel 1.

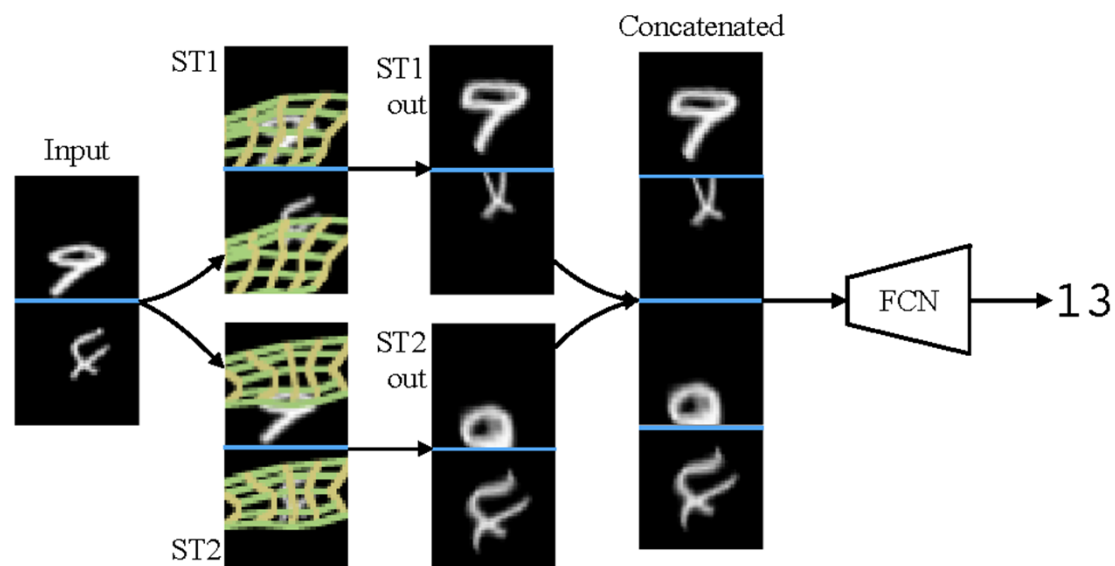
SpatialTransformer2  
**automatically** specialises  
to rectify channel 2.

# Task: Add digits in two images

MNIST digits under rotation, translation and scale

Performance % error

Model		
FCN		47.7
CNN		14.7
ST-FCN	Aff	22.6
	Proj	18.5
	TPS	19.1
2×ST-FCN	Aff	9.0
	Proj	5.9
	TPS	5.8

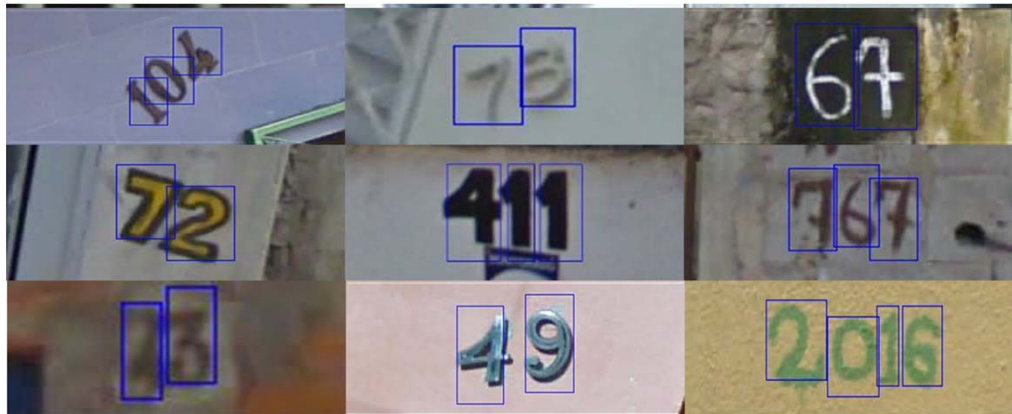




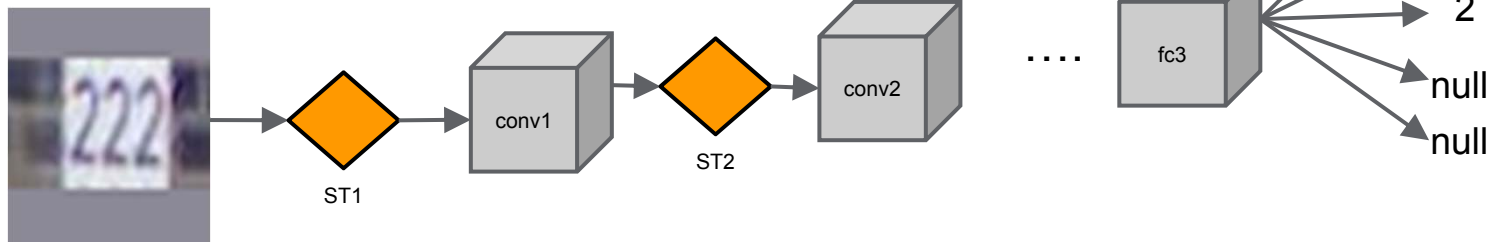
**Applications  
and comparisons with the state of the art**

# Street View House Numbers (SVHN)

200k real images of house numbers collected from Street View  
Between 1 and 5 digits in each number



Architecture:



4 spatial transformer + conv layers, 4 conv layers, 3 fc layers, 5 character output layers

# SVHN 64x64

- **CNN: 4.0% error**

- (single model) Goodfellow et al 2013

- **Attention: 3.9% error**

- (ensemble with MC averaging)

Ba et al, ICLR 2015

- **ST net: 3.6% error**

- (single model)



# SVHN 128x128

- **CNN: 5.6% error**

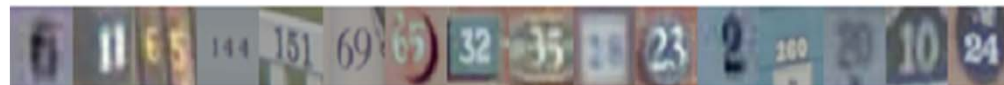
- (single model)



- **Attention: 4.5% error**

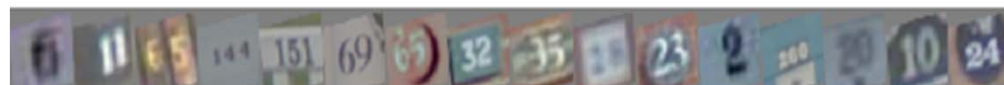
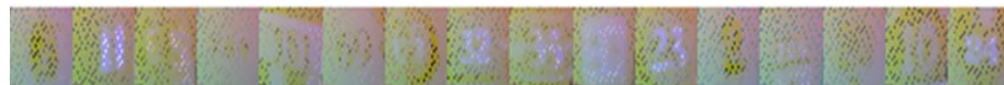
- (ensemble with MC averaging)

Ba et al, ICLR 2015



- **ST net: 3.9% error**

- (single model)



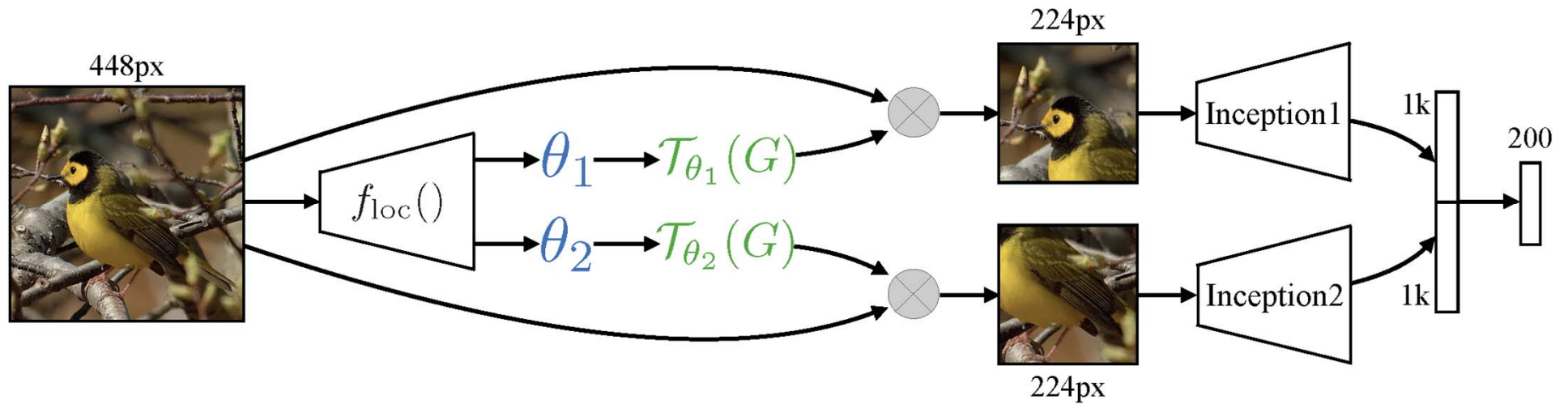
# Fine Grained Visual Categorization

## CUB-200-2011 birds dataset

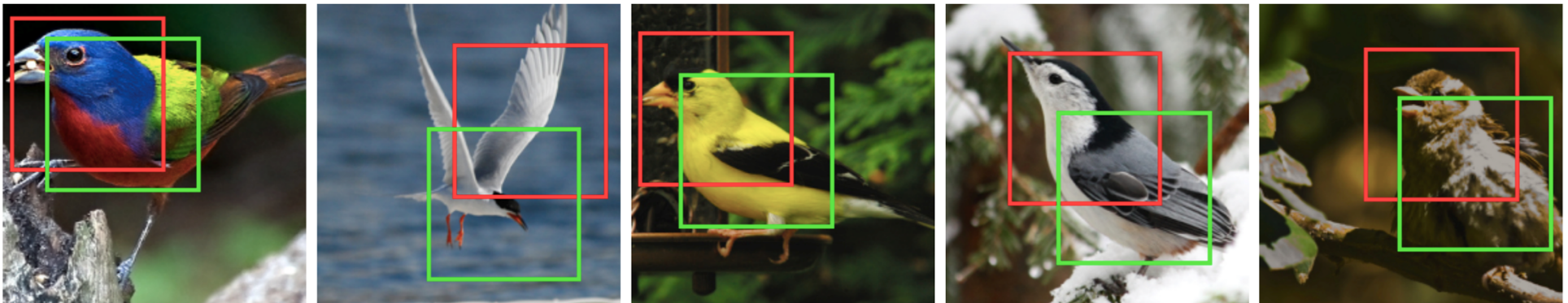
- 200 species of birds
- 6k training images
- 5.8k test images



# Spatial Transformer Network

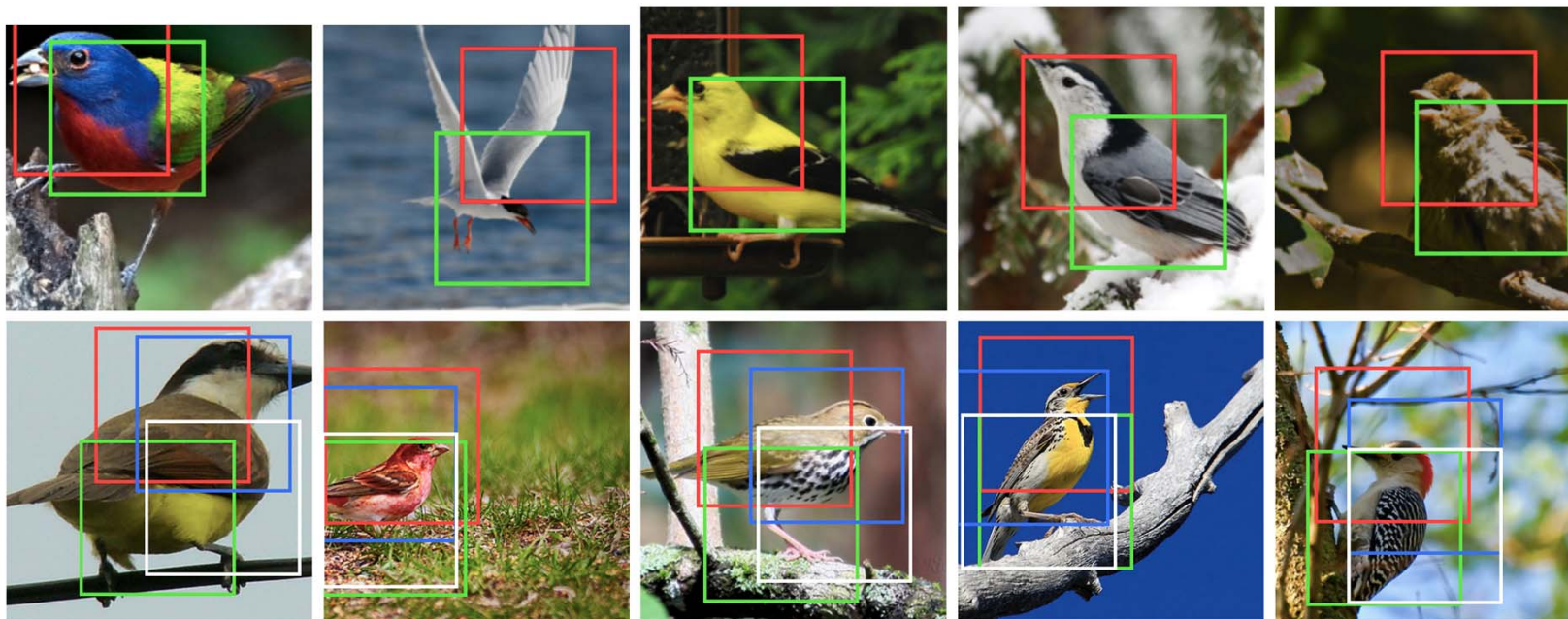


- Pre-train inception networks on ImageNet
- Train spatial transformer network on fine grained multi-way classification



# CUB Performance

Model		
Cimpoi '15 [4]		66.7
Zhang '14 [30]		74.9
Branson '14 [2]		75.7
Lin '15 [20]		80.9
Simon '15 [24]		81.0
CNN (ours) 224px		82.3
2×ST-CNN 224px		83.1
2×ST-CNN 448px		83.9
4×ST-CNN 448px		<b>84.1</b>



# Summary

- Spatial Transformers allow dynamic, conditional cropping and warping of images/feature maps.
- Can be constrained and used as very fast attention mechanism.
- Spatial Transformer Networks localise and rectify objects automatically. Achieve state of the art results.
- Can be used as a generic localisation mechanism which can be learnt with backprop.