Fisher Vector image representation

Machine Learning and Category Representation 2014-2015

Jakob Verbeek, January 9, 2015

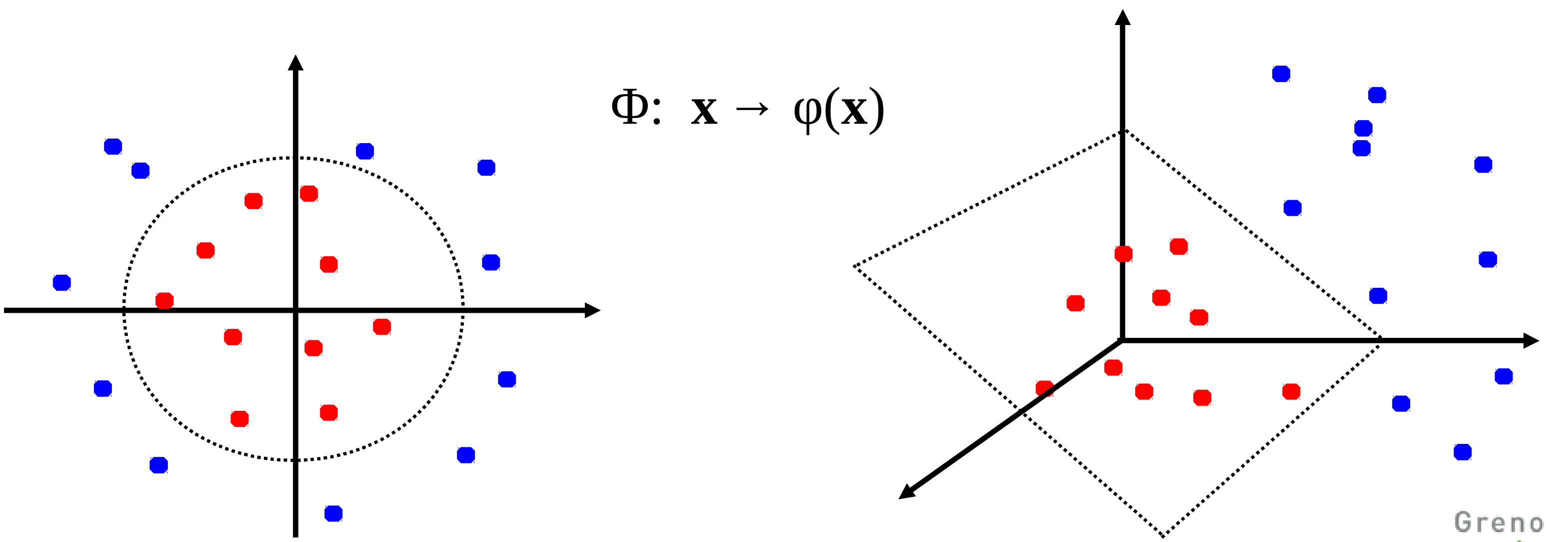
Course website:

http://lear.inrialpes.fr/~verbeek/MLCR.14.15

A brief recap on kernel methods

- A way to achieve non-linear classification by using a kernel that computes inner products of data after non-linear transformation.
  - Given the transformation, we can derive the kernel function.
- Conversely, if a kernel is positive definite, it is known to compute a dot-product in a (not necessarily finite dimensional) feature space.
  - Given the kernel, we can determine the feature mapping function.

$$k(x_{1,} x_2) = \langle \varphi(x_1), \varphi(x_2) \rangle$$



A brief recap on kernel methods

- So far, we considered starting with data in a vector space, and mapping it into another vector space to facilitate linear classification.
- Kernels can also be used to represent non-vectorial data, and to make them amenable to linear classification (or other linear data analysis) techniques.
- For example, suppose we want to classify sets of points in a vector space, where the size of the set can be arbitrarily large.

$$X=\{x_{1,}x_{2,}\dots,x_N\} \quad \text{with} \quad x_i \in R^d$$

- We can define a kernel function that computes the dot-product between representations of sets that are given by the mean and variance of the set of points in each dimension.

$$\varphi(X)=\begin{pmatrix}\text{mean}(X)\\ \text{var}(X)\end{pmatrix}$$

  - Fixed size representation of sets in 2d dimensions
  - Use kernel to compare different sets:

$$k(X_{1,}X_2)=\langle\varphi(X_1),\varphi(X_2)\rangle$$

Fisher kernels

- Proposed by Jaakkola & Haussler, “Exploiting generative models in discriminative classifiers”,In Advances in Neural Information Processing Systems 11, 1998.

- Motivated by the need to represent variably sized objects in a vector space, such as sequences, sets, trees, graphs, etc., such that they become amenable to be used with linear classifiers, and other data analysis tools

- A generic method to define kernels over arbitrary data types based on generative statistical models.
  - Assume we can define a probability distribution over the items we want to represent

$$p(x;\theta),\ \ x \in X\ ,\ \ \theta \in R^D$$

Fisher kernels

- Given a generative data model $p(x;\theta),\ x\in X,\ \theta\in R^D$
- Represent data x in X by means of the gradient of the data log-likelihood, or "Fisher score":
$$g(x)=\nabla_\theta \ln p(x),$$
$$g(x)\in R^D$$
- Define a kernel over X by taking the scaled inner product between the Fisher score vectors: $$k(x,y)=g(x)^T F^{-1} g(y)$$
- Where F is the Fisher information matrix F:
$$F=\boldsymbol{E}_{p(x)}\left[g(x)g(x)^T\right]$$
- Note: the Fisher kernel is a positive definite kernel since
$$k(x_i,x_j)=\left(F^{-1/2}g(x_i)\right)^T\left(F^{-1/2}g(x_j)\right)$$
  - And therefore
$$a^T K a=(Ga)^T Ga\geq 0$$

  where $K_{ij}=k(x_i,x_j)$ and the i-th column of G contains $F^{-1/2}g(x_i)$

Fisher kernels – relation to generative classification

- Suppose we make use of generative model for classification via Bayes' rule
  - Where x is the data to be classified, and y is the discrete class label

$$p(y|x) = p(x|y)\,p(y)/p(x),$$
$$p(x) = \sum_{k=1}^{K} p(y=k)\,p(x|y=k)$$

and

$$p(x|y) = p(x;\theta_y),$$
$$p(y=k) = \pi_k = \frac{\exp(\alpha_k)}{\sum_{k'=1}^{K} \exp(\alpha_{k'})}$$

- Classification with the Fisher kernel obtained using the marginal distribution p(x) is at least as powerful as classification with Bayes' rule.
- This becomes useful when the class conditional models are poorly estimated, either due to bias or variance type of errors.
- In practice often used without class-conditional models, but direct generative model for the marginal distribution on X.

Fisher kernels – relation to generative classification

- Consider the Fisher score vector with respect to the marginal distribution on X

$$\nabla_\theta \ln p(x) = \frac{1}{p(x)} \nabla_\theta \sum_{k=1}^{K} p(x, y=k)$$

$$= \frac{1}{p(x)} \sum_{k=1}^{K} p(x, y=k) \nabla_\theta \ln p(x, y=k)$$

$$= \sum_{k=1}^{K} p(y=k|x) \left[\nabla_\theta \ln p(y=k) + \nabla_\theta \ln p(x|y=k) \right]$$

- In particular for the alpha that model the class prior probabilities we have

$$\frac{\partial \ln p(x)}{\partial \alpha_k} = p(y=k|x) - \pi_k$$

Fisher kernels – relation to generative classification

$$\frac{\partial \ln p(x)}{\partial \alpha_k} = p(y=k|x) - \pi_k$$

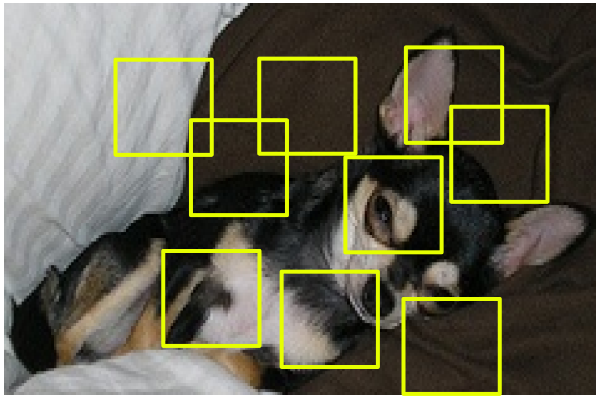
$$g(x) = \nabla_\theta \ln p(x) = \left(\frac{\partial \ln p(x)}{\partial \alpha_1}, \ldots, \frac{\partial \ln p(x)}{\partial \alpha_K}, \ldots \right)$$

- Consider discriminative multi-class classifier.
- Let the weight vector for the k-th class to be zero, except for the position that corresponds to the alpha of the k-th class where it is one. And let the bias term for the k-th class be equal to the prior probability of that class,
- Then $f_k(x) = w_k^T g(x) + b_k = p(y=k|x)$

  and thus $\operatorname{argmax}_k f_k(x) = \operatorname{argmax}_k p(y=k|x)$
- Thus the Fisher kernel based classifier can implement classification via Bayes' rule, and generalizes it to other classification functions.

Local descriptor based image representations

- Patch extraction and description stage
  - For example: SIFT, HOG, LBP, color, ...
  - Dense multi-scale grid, or interest points



$$X=\{x_{1,}\dots,x_N\}$$

- Coding stage: embed local descriptors, typically in higher dimensional space
  - For example: assignment to cluster indices

$$\varphi(x_i)$$

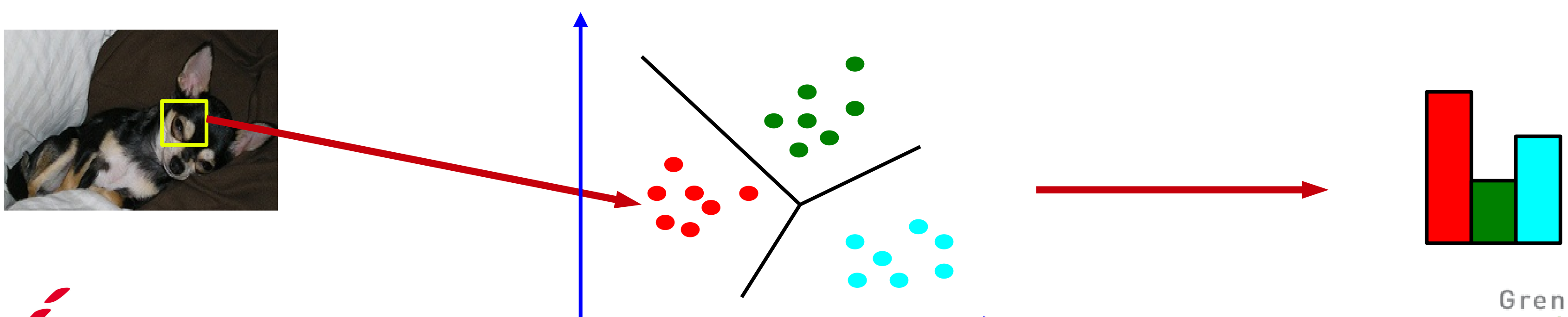
- Pooling stage: aggregate per-patch embeddings
  - For example: sum pooling

$$\Phi(X)=\sum_{i=1}^{N}\varphi(x_i)$$

Bag-of-word image representation

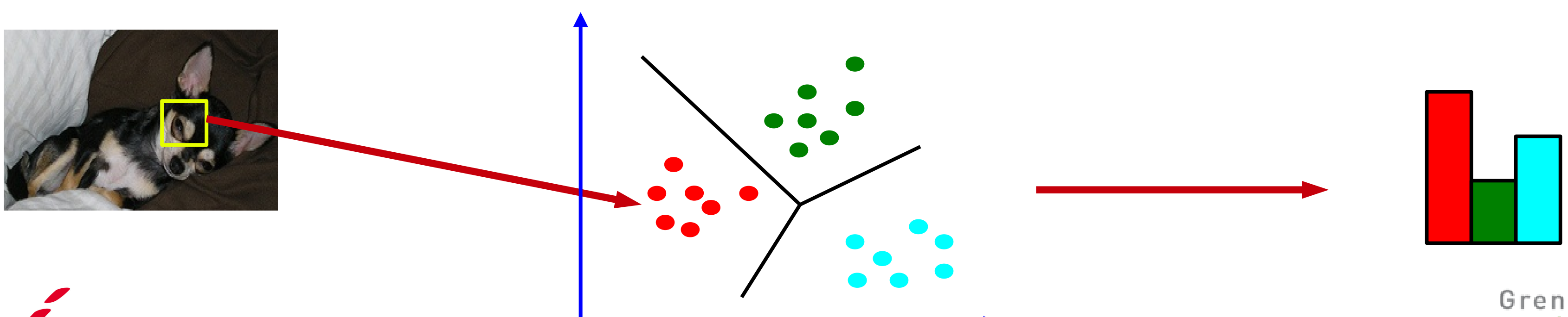
- Extract local image descriptors, e.g. SIFT
  - Dense on multi-scale grid, or on interest points
- Off-line: cluster local descriptors with k-means
  - Using random subset of patches from training images
- To represent training or test image
  - Assign SIFTs to cluster indices / visual words $\varphi(x_i) = [0, \ldots, 0, 1, 0, \ldots, 0]$
  - Histogram of cluster counts aggregates all local feature information

[Sivic & Zisserman, ICCV'03], [Csurka et al., ECCV'04] $h = \sum_i \varphi(x_i)$



Application of FV for bag-of-words image-representation

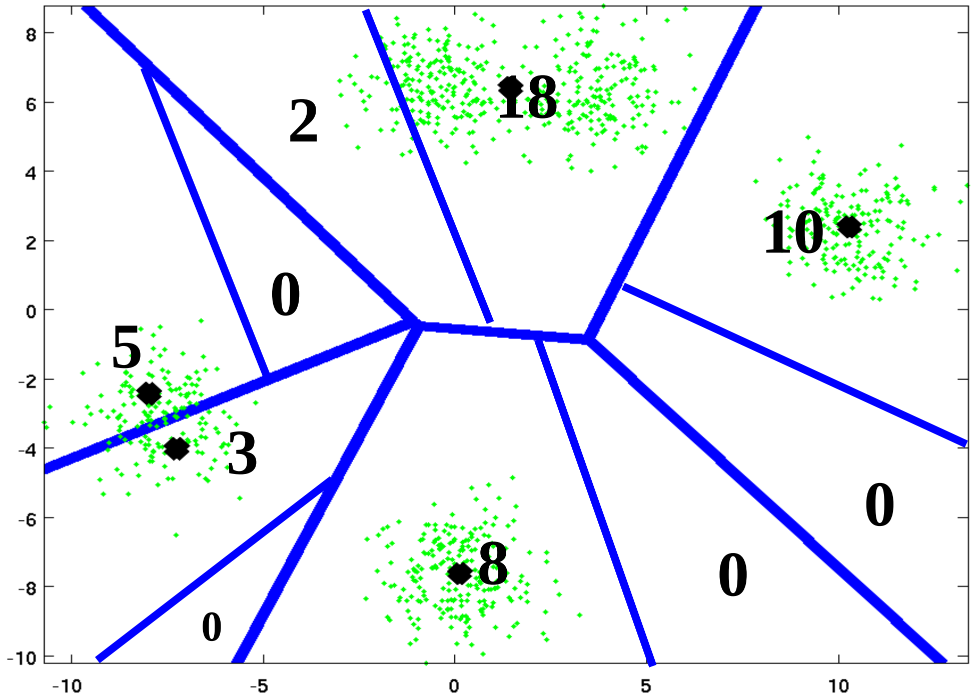
- Bag of word (BoW) representation
  - Map every descriptor to a cluster / visual word index $w_i \in \{1,\ldots,K\}$
- Model visual word indices with i.i.d. multinomial $p(w_i=k)=\frac{\exp \alpha_k}{\sum_{k'} \exp \alpha_{k'}}=\pi_k$
  - Likelihood of N i.i.d. indices: $p(w_{1:N})=\prod_{i=1}^{N} p(w_i)$
  - Fisher vector given by gradient $\frac{\partial \ln p(w_{1:N})}{\partial \alpha_k}=\sum_{i=1}^{N} \frac{\partial \ln p(w_i)}{\partial \alpha_k}=h_k - N\pi_k$
    - i.e. BoW histogram + constant



Fisher vector GMM representation: Motivation

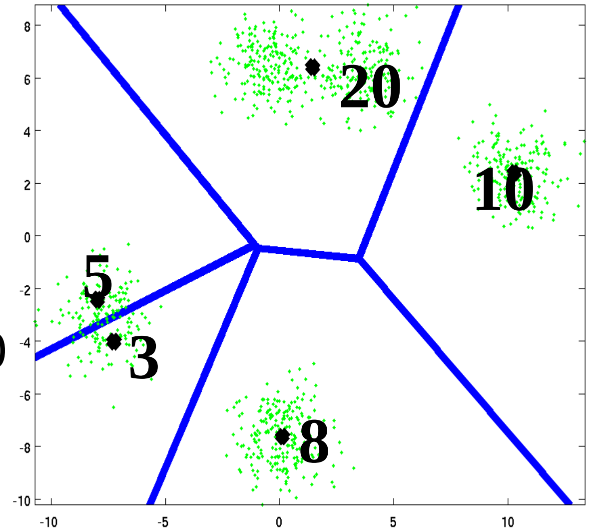
- Suppose we want to refine a given visual vocabulary to obtain a richer image representation

- Bag-of-word histogram stores # patches assigned to each word
  - Need more words to refine the representation
  - But this directly increases the computational cost
  - And leads to many empty bins: redundancy



Fisher vector GMM representation: Motivation

- Feature vector quantization is computationally expensive
- To extract visual word histogram for a new image
  - Compute distance of each local descriptor to each k-means center
  - run-time O(NKD) : linear in
    - N: nr. of feature vectors ~ 10^4 per image
    - K: nr. of clusters ~ 10^3 for recognition
    - D: nr. of dimensions ~ 10^2 (SIFT)



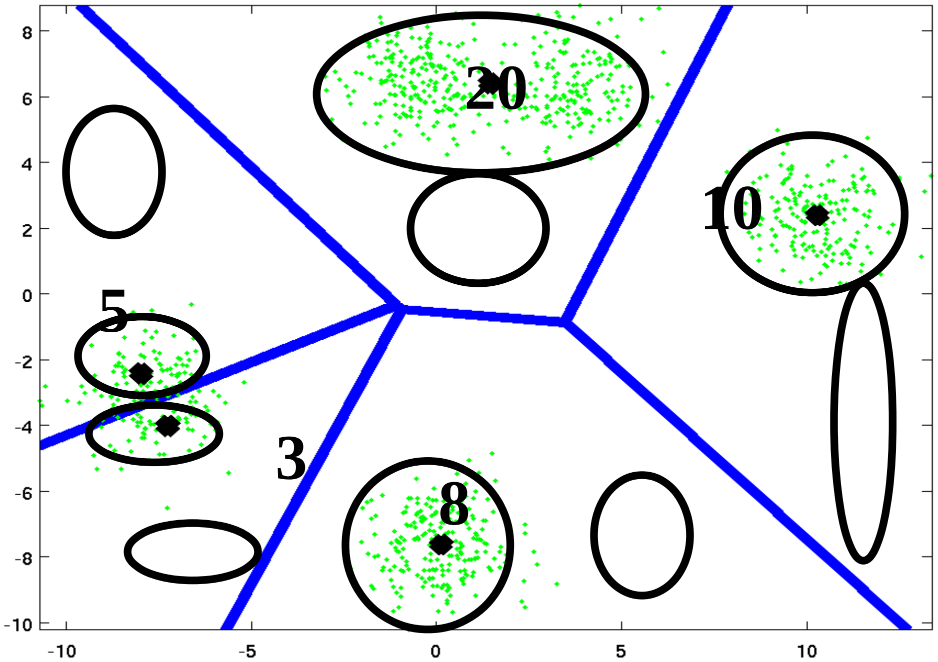
- So in total in the order of 10^9 multiplications per image to obtain a histogram of size 1000
- Can this be done more efficiently ?!
  - Yes, extract more than just a visual word histogram from a given clustering

Fisher vector representation in a nutshell

- Instead, the Fisher Vector for GMM also records the mean and variance of the points per dimension in each cell
  - More information for same # visual words
  - Does not increase computational time significantly
  - Leads to high-dimensional feature vectors

- Even when the counts are the same,

  the position and variance of the points in the cell can vary



Application of FV for Gaussian mixture model of local features

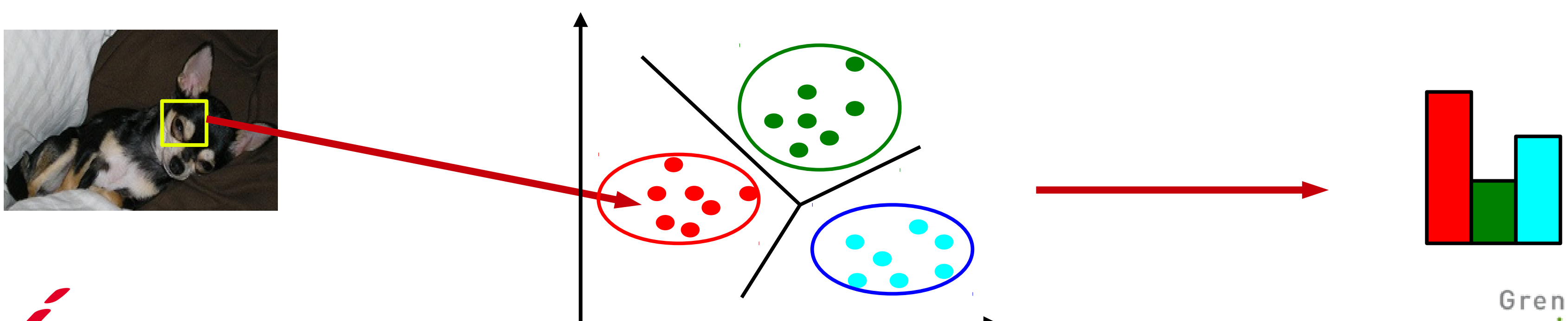
- Gaussian mixture models for local image descriptors

  [Perronnin & Dance, CVPR 2007]
  - State-of-the-art feature pooling for image/video classification/retrieval

- Offline: Train k-component GMM on collection of local features

$$p(x)=\sum_{k=1}^{K}\pi_k N(x;\mu_k,\sigma_k)$$

- Each mixture component corresponds to a visual word
  - Parameters of each component: mean, variance, mixing weight
  - We use diagonal covariance matrix for simplicity
    - Coordinates assumed independent, per Gaussian



Application of FV for Gaussian mixture model of local features

- Gaussian mixture models for local image descriptors
  [Perronnin & Dance, CVPR 2007]
  - State-of-the-art feature pooling for image/video classification/retrieval

- Representation: gradient of log-likelihood
  - For the means and variances we have:

$$F^{-1/2}\nabla_{\mu_k}\ln p(x_{1:N})=\frac{1}{\sqrt{\pi_k}}\sum_{n=1}^{N} p(k|x_n)\frac{(x_n-\mu_k)}{\sigma_k}$$

$$F^{-1/2}\nabla_{\sigma_k}\ln p(x_{1:N})=\frac{1}{\sqrt{2\pi_k}}\sum_{n=1}^{N} p(k|x_n)\left\{\frac{(x_n-\mu_k)^2}{\sigma_k^2}-1\right\}$$

  - Soft-assignments given by component posteriors

$$p(k|x_n)=\frac{\pi_k N(x_n;\mu_k,\sigma_k)}{p(x_n)}$$

Application of FV for Gaussian mixture model of local features

- Fisher vector components give the difference between the data mean predicted by the model and observed in the data, and similar for variance.

- For the gradient w.r.t. the mean

$$F^{-1/2}\nabla_{\mu_k}\ln p(x_{1:N}) = \frac{1}{\sqrt{\pi_k}}\sum_{n=1}^{N} p(k|x_n)\frac{(x_n-\mu_k)}{\sigma_k} = \frac{n_k}{\sigma_k\sqrt{\pi_k}}(\hat{\mu}_k-\mu_k)$$

  - where $n_k = \sum_{n=1}^{N} p(k|x_n)$ $\hat{\mu}_k = n_k^{-1}\sum_{n=1}^{N} p(k|x_n)\,x_n$

- Similar for the gradient w.r.t. the variance

$$F^{-1/2}\nabla_{\sigma_k}\ln p(x_{1:N}) = \frac{1}{\sqrt{2\pi_k}}\sum_{n=1}^{N} p(k|x_n)\left\{\frac{(x_n-\mu_k)^2}{\sigma_k^2}-1\right\} = \frac{n_k}{\sigma_k^2\sqrt{2\pi_k}}(\hat{\sigma}_k^2-\sigma_k^2)$$

  - where $\hat{\sigma}_k^2 = n_k^{-1}\sum_{n=1}^{N} p(k|x_n)(x_n-\mu_k)^2$

Image representation using Fisher kernels

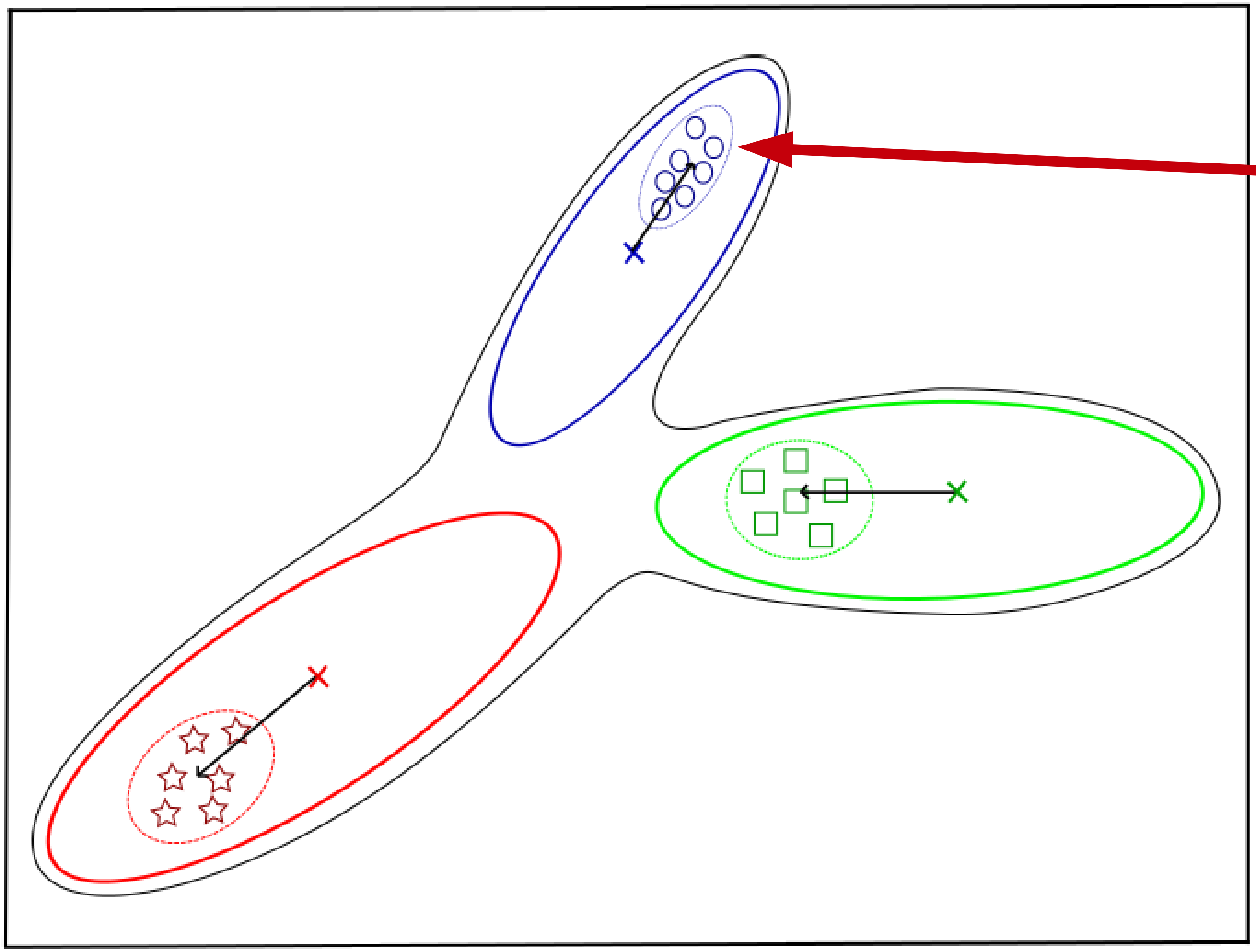
- Data representation

$$G(X,\Theta) = F^{-1/2} \left(\frac{\partial L}{\partial \alpha_1}, \; \ldots \; , \frac{\partial L}{\partial \alpha_K} \; , \; \nabla_{\mu_1} L, \; \ldots \; , \nabla_{\mu_K} L \; , \; \nabla_{\sigma_1} L, \; \ldots \; , \; \nabla_{\sigma_K} L \right)^T$$

- In total K(1+2D) dimensional representation, since for each visual word / Gaussian we have
  - Mixing weight (1 scalar)
  - Mean (D dimensions)
  - Variances (D dimensions, since single variance per dimension)

- Gradient with respect to mixing weights often dropped in practice since it adds little discriminative information for classification.
  - Results in 2KD dimensional image descriptor

Illustration of gradient w.r.t. means of Gaussians



New Data Points

BoW and FV from a function approximation viewpoint

- Let us consider uni-dimensional descriptors: vocabulary quantizes real line
- For both BoW and FV the representation of an image is obtained by sum-pooling the representations of descriptors.
  - Ensemble of descriptors sampled in an image $X=\{x_{1,}\ldots,x_N\}$
  - Representation of single descriptor
    - One-of-k encoding for BoW $\varphi(x_i)=[0,\ldots,0,1,0,\ldots,0]$
    - For FV concatenate per-visual word gradients of form

$$\varphi(x_i)=\left(\ldots,p(k|x_i)\left[1 \quad \frac{(x_i-\mu_k)}{\sigma_k} \quad \frac{(x_i-\mu_k)^2-\sigma_k^2}{\sigma_k^2}\right],\ldots\right)$$

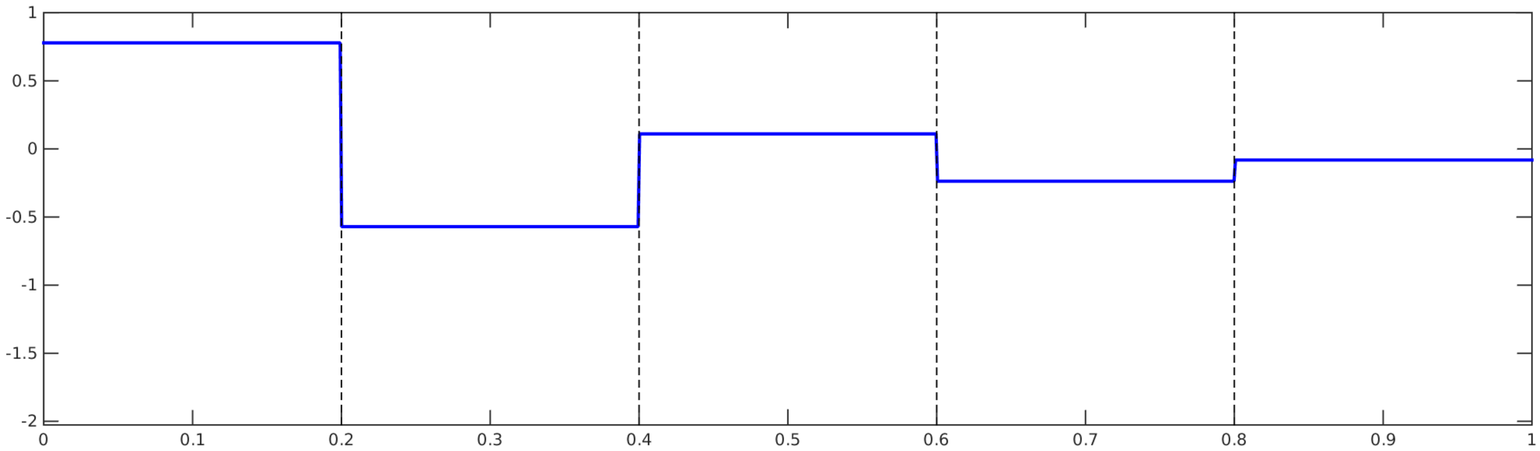
- Linear function of sum-pooled descriptor encodings is a sum of linear functions of individual descriptor encodings:

$$\Phi(X)=\sum_{i=1}^{N}\varphi(x_i)$$

$$w^T\Phi(X)=\sum_{i=1}^{N}w^T\varphi(x_i)$$

From a function approximation viewpoint

- Consider the score of a single descriptor for BoW
  - If assigned to k-th visual word then $w^T \varphi(x_i) = w_k$
  - Thus: constant score for all descriptors assigned to a visual word



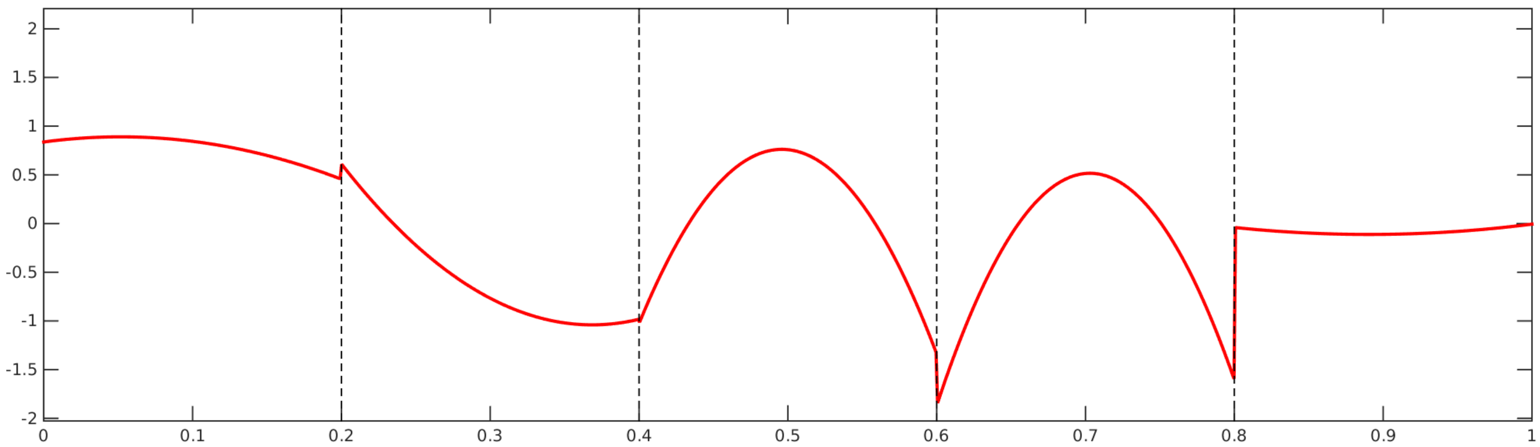
Each cell corresponds to a visual word

From a function approximation viewpoint

- Consider the same for FV, and assume soft-assignment is “hard”
  - Thus: assume for one value of k we have $p(k|x_i) \approx 1$
  - If assigned to the k-th visual word:

$$w^T \varphi(x_i) = w_k^T \left[1 \quad \frac{(x_i - \mu_k)}{\sigma_k} \quad \frac{(x_i - \mu_k)^2 - \sigma_k^2}{\sigma_k^2} \right]$$

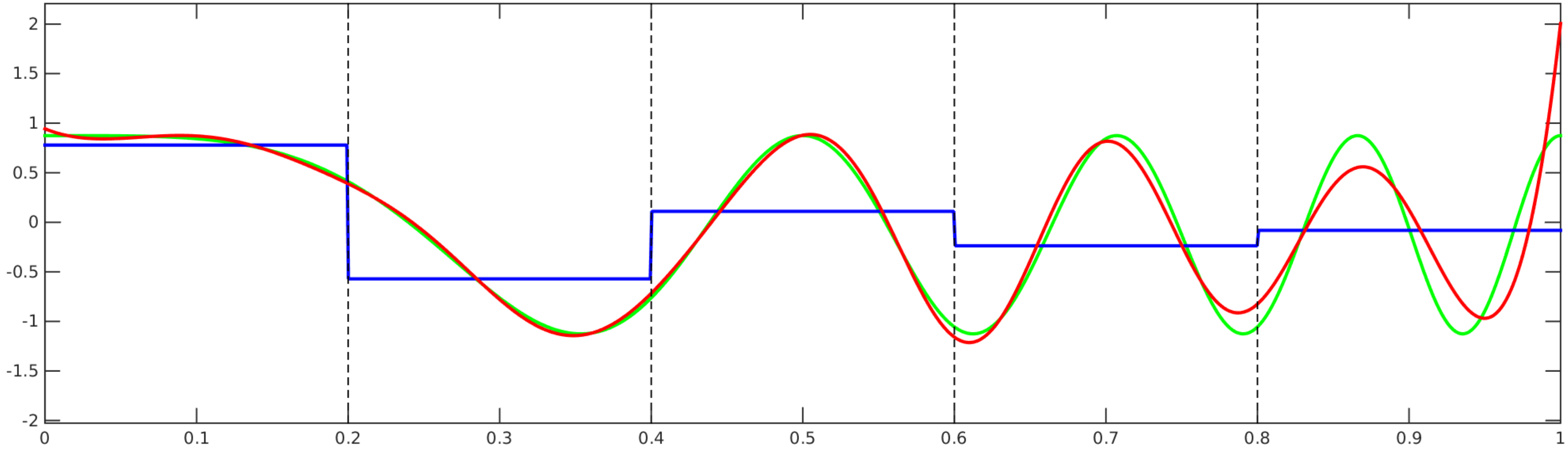
    - Note that w_k is no longer a scalar but a vector
  - Thus: score is a second-order polynomial of the descriptor x, for descriptors assigned to a given visual word.



From a function approximation viewpoint

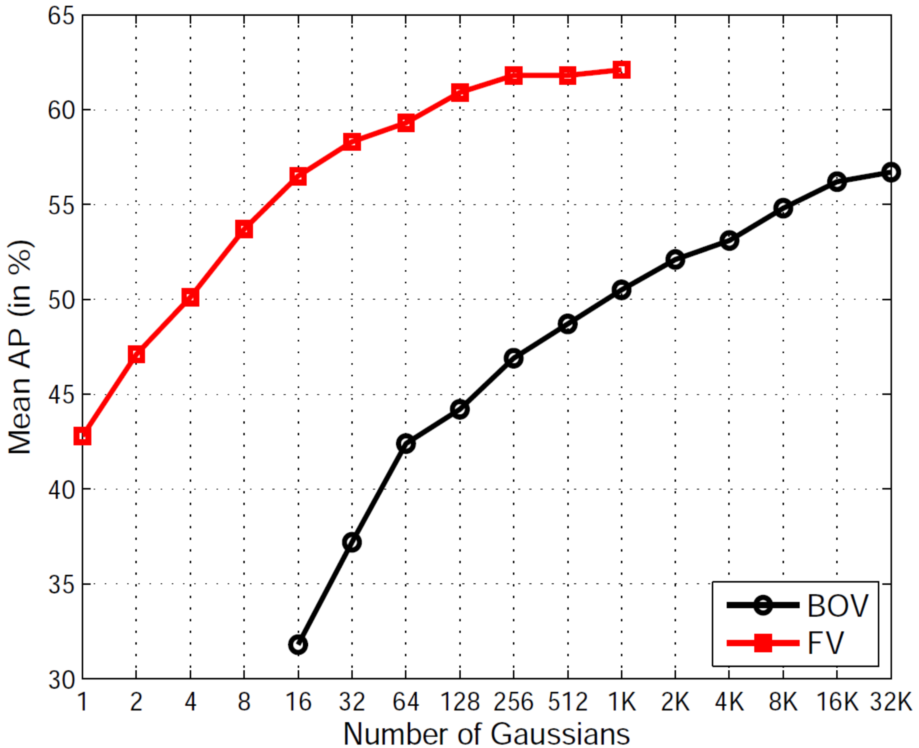
- Consider that we want to approximate a true classification function (green) based on either BoW (blue) or FV (red) representation
  - Weights for BoW and FV representation fitted by least squares to optimally match the target function

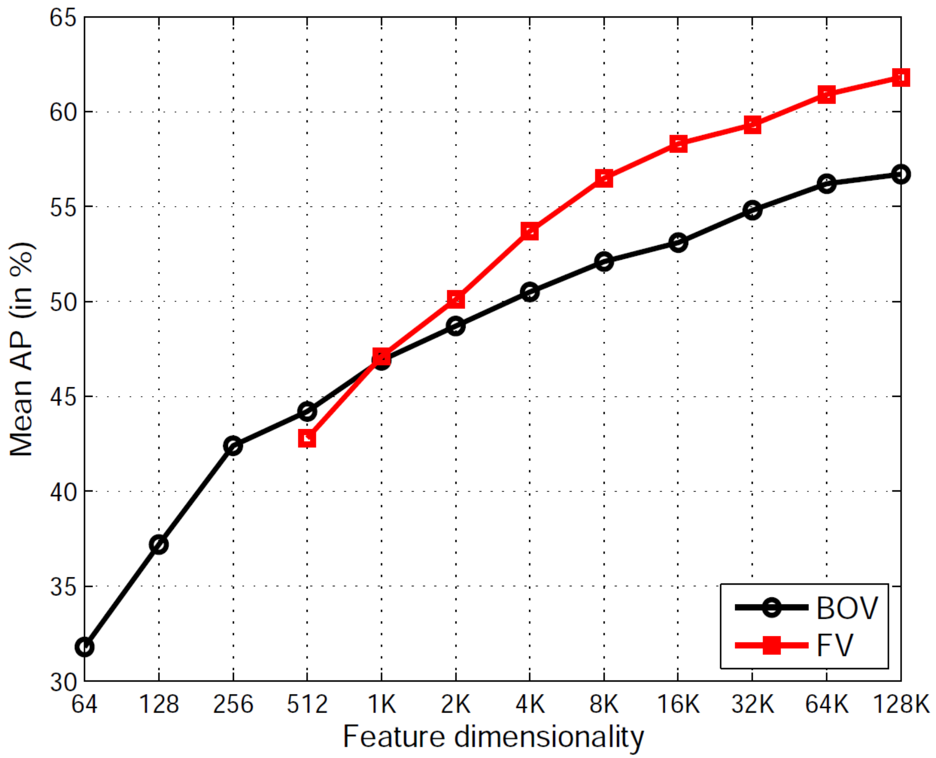
- Better approximation with FV
  - Local second order approximation, instead of local zero-order
  - Smooth transition from one visual word to the next



Fisher vectors: classification performance VOC'07

- Fisher vector representation yields better performance for a given number of Gaussians / visual words than Bag-of-words.
- For a fixed dimensionality Fisher vectors perform better, and are more efficient to compute





Normalization of the Fisher vector

- Inverse Fisher information matrix *F* $F = E[g(x)g(x)^T]$
  - Renders FV invariant for re-parametrization $f(x) = F^{-1/2} g(x)$
  - Linear projection, analytical approximation for MoG gives diagonal matrix

    [Jaakkola, Haussler, NIPS 1999], [Sanchez, Perronnin, Mensink, Verbeek IJCV'13]

- Power-normalization $f(x) \leftarrow sign(f(x))|f(x)|^{\rho}$
  - Renders Fisher vector less sparse $0 < \rho < 1$

    [Perronnin, Sanchez, Mensink, ECCV'10]
  - Corrects for poor independence assumption on local descriptors

    [Cinbis, Verbeek, Schmid, CVPR'12]

- L2-normalization
  - Makes representation invariant to number of local features $f(x) \leftarrow \frac{f(x)}{\sqrt{f(x)^T f(x)}}$
  - Among other Lp norms the most effective with linear classifier

    [Sanchez, Perronnin, Mensink, Verbeek IJCV'13]

Normalization with inverse Fisher information matrix

- Gradient of log-likelihood w.r.t. parameters $g(x)=\nabla_\theta \ln p(x)$
- Fisher information matrix $F_\theta=\int g(x)g(x)^T p(x)dx$
- Normalized Fisher kernel $k(x_{1,}x_2)=g(x_1)^T F_\theta^{-1} g(x_2)$

- Consider different parametrization given by some invertible function $\lambda=f(\theta)$
- Jacobian matrix relating the parametrizations $[J]_{ij}=\frac{\partial\theta_j}{\partial\lambda_i}$
- Gradient of log-likelihood w.r.t. new parameters

$$h(x)=\nabla_\lambda \ln p(x)=J\nabla_\theta \ln p(x)=J\,g(x)$$

- Fisher information matrix $F_\lambda=\int h(x)h(x)^T p(x)dx=J F_\theta J^T$
- Normalized Fisher kernel

$$\begin{aligned} h(x_1)^T F_\lambda^{-1} h(x_2) &= g(x_1)^T J^T (JF_\theta J^T)^{-1} J\,g(x_2) \\ &= g(x_1)^T J^T J^{-T} F_\theta^{-1} J^{-1} J\,g(x_2) \\ &= g(x_1)^T F_\theta^{-1} g(x_2) \\ &= k(x_{1,}x_2) \end{aligned}$$

Effect of power and L2 normalization in practice

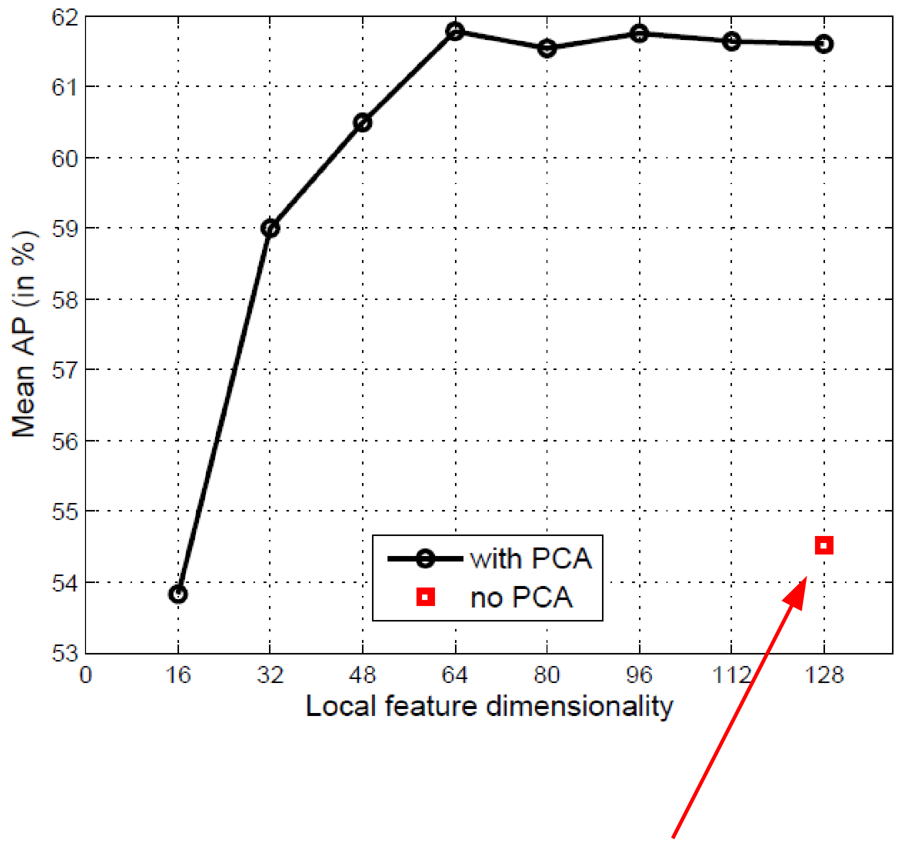
- Classification results on the PASCAL VOC 2007 benchmark dataset.
- Regular dense sampling of local SIFT descriptors in the image
  - PCA projected to 64 dimensions
- Using mixture of 256 Gaussians over the SIFT descriptors
  - FV dimensionality: 2*64*256 = 32 * 1024

Power Nomalization	L2 normalization	Performance (mAP)	Improvement over baseline
No	No	51.5	0
Yes	No	59.8	8.3
No	Yes	57.3	5.8
Yes	Yes	61.8	10.3

PCA dimension reduction of local descriptors

- We use diagonal covariance model
- Dimensions might be correlated
- Apply PCA projection to
  - De-correlate features
  - Reduce dimension of final FV
- FV with 256 Gaussians over local SIFT descriptors of dimension 128

Results on PASCAL VOC’07:



Example applications: Fine-grained classification

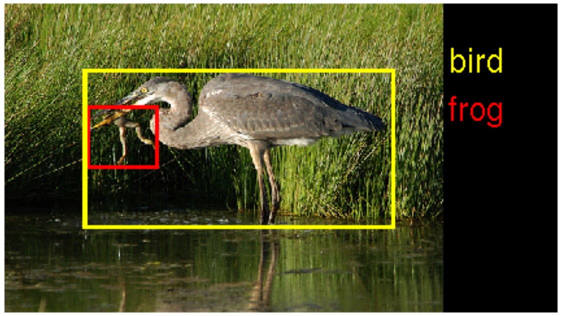
aircraft (100) birds (83) cars (196) dogs (120) shoes (70)

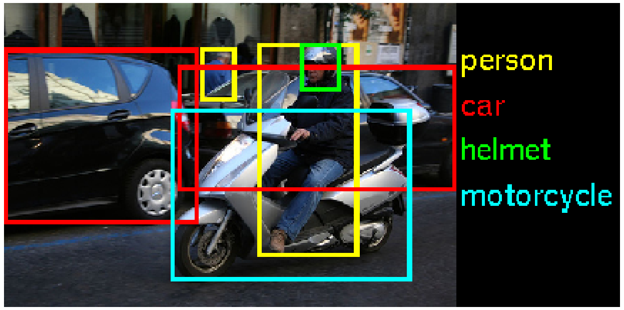
- Winning INRIA+Xerox system at FGComp'13: http://sites.google.com/site/fgcomp2013
  - multiple low-level descriptors: SIFT, color, etc.
  - Fisher Vector embedding

    Gosselin, Murray, Jégou, Perronnin, "Revisiting the Fisher vector for fine-grained classification", PRL'14.

- Many other successful uses of FVs for fine-grained recognition
  - Rodriguez and Larlus, "Predicting an object location using a global image representation", ICCV'13.
  - Gavves, Fernando, Snoek, Smeulders, Tuytelaars, "Fine-Grained Categorization by Alignments", ICCV'13
  - Chai, Lempitsky, Zisserman, "Symbiotic segmentation and part localization for fine-grained categorization", ICCV'13
  - Murray, Perronnin, "Generalized Max Pooling", CVPR'14.

Example applications: object detection





- ImageNet’13 detection: http://www.image-net.org/challenges/LSVRC/2013/
- Winning system by University of Amsterdam
  - region proposals with selective search
  - Fisher Vector embedding
  - Fast Local Area Independent Representation (FLAIR)

Van de Sande, Snoek, Smeulders, “Fisher and VLAD with FLAIR”, CVPR’14.

Example applications: face verification

- Face track description:
  - track face
  - extract SIFT descriptors
  - encode using Fisher vectors
  - pool at face track level

Parkhi, Simonyan, Veldaldi, Zisserman, "A compact and discriminative face track descriptor", CVPR'14.

- New state-of-the-art results on the YouTube faces dataset

	Method	Accuracy	AUC	EER
1	MGBS & SVM- [37]	78.9 ± 1.9	86.9	21.2
2	APEM FUSION [20]	79.1 ± 1.5	86.6	21.4
3	STFRD & PMML [11]	79.5 ± 2.5	88.6	19.9
4	VSOF & OSS (Adaboost) [22]	79.7 ± 1.8	89.4	20.0
5	Our VF^2 (restricted)	83.5 ± 2.3	92.0	16.1
6	Our VF^2 (restricted & flip)	84.7 ± 1.4	93.0	14.9
7	Our VF^2 (unrestricted & flip)	83.5 ± 2.1	94.0	13.0
8	Our VF^2 (unrestricted & jitt. pool.)	83.8 ± 1.6	95.0	**12.3**

Example applications: action recognition and localization

- THUMOS action recognition challenge 2013 & 2014

  http://crcv.ucf.edu/ICCV13-Action-Workshop

- Winning systems by INRIA-LEAR
  - improved dense trajectory video features
  - Fisher Vector embedding

Wang and Schmid, “Action Recognition with Improved Trajectories”, ICCV’13.

Bag-of-words vs. Fisher vector image representation

- GMM Fisher vector is an alternative to bag-of-words image representation introduced in
  - *Fisher kernels on visual vocabularies for image categorization*
    F. Perronnin and C. Dance, CVPR 2007.
- Both representations based on a visual vocabulary obtained by means of clustering local descriptors
- Bag-of-words image representation
  - Off-line: fit k-means clustering to local descriptors
  - Represent image with histogram of visual word counts: K dimensions
- Fisher vector image representation
  - Off-line: fit GMM model to local descriptors
  - Represent image with gradient of log-likelihood: K(2D+1) dimensions

Summary of Fisher vector image representation

- Computational cost similar:
  - Both compare N descriptors to K clusters (visual words)
- Memory usage:
  - Fisher vector has size 2KD for K clusters and D dim. descriptors
  - Bag-of-word has size K for K clusters
- For a given dimension of the representation
  - FV needs less clusters, and is faster to compute
  - FV gives better performance since it is a smoother function of the local descriptors.

- A recent overview article on Fisher Vector representation
  - Image Classification with the Fisher Vector: Theory and Practice
    Jorge Sanchez; Florent Perronnin; Thomas Mensink; Jakob Verbeek
    International Journal of Computer Vision, Springer, 2013

informatics mathematics

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