#### Fisher vector image representation

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Course website: http://lear.inrialpes.fr/~verbeek/MLCR.11.12.php

#### Fisher vector representation

 Alternative to bag-of-words image representation introduced in Fisher kernels on visual vocabularies for image categorization F. Perronnin and C. Dance, CVPR 2007.

- FV in comparison to the BoW representation
  - Both FV and BoW are based on a visual vocabulary, with assignment of patches to visual words
  - FV based on Mixture of Gaussian clustering of patches, BoW based on k-means clustering
  - FV Extracts a larger image signature than the BoW representation for a given number of visual words
  - Leads to good classification results using linear classifiers, where BoW representations require non-linear classifiers.

## Fisher vector representation: Motivation 1

- Suppose we use a bag-of-words image representation
  - Visual vocabulary trained offline
- Feature vector quantization is computationally expensive in practice
- To extract visual word histogram for a new image
  - Compute distance of each local descriptor to each k-means center
  - run-time O(NKD) : linear in
    - N: nr. of feature vectors ~ 10^4 per image
    - K: nr. of clusters ~ 10^3 for recognition
    - D: nr. of dimensions ~ 10^2 (SIFT)
- So in total in the order of 10^9 multiplications per image to obtain a histogram of size 1000
- Can this be done more efficiently ?!
  - Yes, extract more than just a visual word histogram !



### Fisher vector representation: Motivation 2

- Suppose we want to refine a given visual vocabulary
- Bag-of-word histogram stores # patches assigned to each word
  - Need more words to refine the representation
  - But this directly increases the computational cost
  - And leads to many empty bins, redundancy



### Fisher vector representation: Motivation 2

- Instead, the Fisher Vector also records the mean and variance of the points per dimension in each cell
  - More information for same # visual words
  - Does not increase computational time significantly
  - Leads to high-dimensional feature vectors
- Even when the counts are the same the position and variance of the points in the cell can vary



#### Image representation using Fisher kernels

- General idea of Fischer vector representation
  - Fit probabilistic model to data  $p(X; \Theta)$
  - Represent data with derivative of data log-likelihood "How does the data want that the model changes?"  $G(X,\Theta) = \frac{\partial \log p(x;\Theta)}{\partial \Theta}$

Jaakkola & Haussler. "Exploiting generative models in discriminative classifiers", in Advances in Neural Information Processing Systems 11, 1999.

• We use Mixture of Gaussians to model the local (SIFT) descriptors  $X = \{x_n\}_{n=1}^N$   $L(X, \Theta) = \sum_n \log p(x_n)$  $p(x_n) = \sum_k \pi_k N(x_n; m_k, C_k)$ 

 Define mixing weights using the soft-max function ensures positiveness and sum to one constraint

$$\pi_k = \frac{\exp \alpha_k}{\sum_{k'} \exp \alpha_{k'}}$$

#### Image representation using Fisher kernels

- Mixture of Gaussians to model the local (SIFT) descriptors  $L(\Theta) = \sum_{n} \log p(x_{n})$   $p(x_{n}) = \sum_{k} \pi_{k} N(x_{n}; m_{k}, C_{k})$ 
  - The parameters of the model are  $\Theta$

$$\Theta = \{\alpha_k, m_k, C_k\}_{k=1}^K$$

- where we use diagonal covariance matrices

Concatenate derivatives to obtain data representation

$$G(X,\Theta) = \left(\frac{\partial L}{\partial \alpha_1}, \dots, \frac{\partial L}{\partial \alpha_K}, \frac{\partial L}{\partial m_1}, \dots, \frac{\partial L}{\partial m_K}, \frac{\partial L}{\partial C_1^{-1}}, \dots, \frac{\partial L}{\partial C_K^{-1}}\right)^T$$

#### Image representation using Fisher kernels

Data representation

$$G(X,\Theta) = \left(\frac{\partial L}{\partial \alpha_1}, \dots, \frac{\partial L}{\partial \alpha_K}, \frac{\partial L}{\partial m_1}, \dots, \frac{\partial L}{\partial m_K}, \frac{\partial L}{\partial C_1^{-1}}, \dots, \frac{\partial L}{\partial C_K^{-1}}\right)^T$$

In total K(1+2D) dimensional representation, ۲ since for each visual word / Gaussian we have

Count (1 dim): 
$$\frac{\partial L}{\partial \alpha_k} = \sum_n (q_{nk} - \pi_k)$$
  
Mean (D dims): 
$$\frac{\partial L}{\partial \alpha_k} = C_k^{-1} \sum_{k=1}^{n} q_{nk} (x_n - m_k)$$
  
Center of assigned data  
Relative to cluster center

 $\frac{\partial m_k}{\partial m_k} = C_k \sum_n q_{nk} (x_n - m_k)$ 

Relative to cluster cente

Variance (D dims):  $\frac{\partial L}{\partial C_{k}^{-1}} = \frac{1}{2} \sum_{n} q_{nk} (C_{k} - (x_{n} - m_{k})^{2})$  Variance of assigned data relative to cluster variance

With the soft-assignments:

$$q_{nk} = p(k|x_n) = \frac{\pi_k p(x_n|k)}{p(x_n)}$$

#### Bag-of-words vs. Fisher vector image representation

- Bag-of-words image representation
  - Off-line: fit k-means clustering to local descriptors
  - Represent image with histogram of visual word counts: K dimensions
- Fischer vector image representation
  - Off-line: fit MoG model to local descriptors
  - Represent image with derivative of log-likelihood: K(2D+1) dimensions
- Computational cost similar:
  - Both compare N descriptors to K visual words (centers / Gaussians)
- Memory usage: higher for fisher vectors
  - Fisher vector is a factor (2D+1) larger, e.g. a factor 257 for SIFTs !
    - Ie for 1000 visual words this is roughly 257\*1000\*4 bytes ~ 1 Mb
  - However, because we store more information per visual word, we can generally obtain same or better performance with far less visual words

#### Images from categorization task PASCAL VOC

Yearly evaluation since 2005 for image classification (also object localization, segmentation, and body-part localization)













Cat























Person





Sheep





#### Fisher vectors: classification performance

- Results taken from: "Fisher Kernels on Visual Vocabularies for Image Categorization", F. Perronnin and C. Dance, in CVPR '07
- BoW and Fisher vector yield similar performance
  - Fisher vector uses 32x fewer Gaussians
  - BoW representation 2.000 long, FV length is  $64(1+2 \times 128) = 16.448$



# Additional reading material

- Fisher vector image representation
  - "Fisher Kernels on Visual Vocabularies for Image Categorization"
    - F. Perronnin and C. Dance, in CVPR '07
- Pattern Recognition and Machine Learning Chris Bishop, 2006, Springer
  - Section 6.2

# Exam

- Friday January 27<sup>th</sup>
  - From 9 am to 12 am
  - Room H105 Ensimag building @ campus
- Prepare from
  - Lecture slides
  - Presented papers
  - Bishop's book
- During the exam you can bring
  - the lecture slides
  - the presented papers