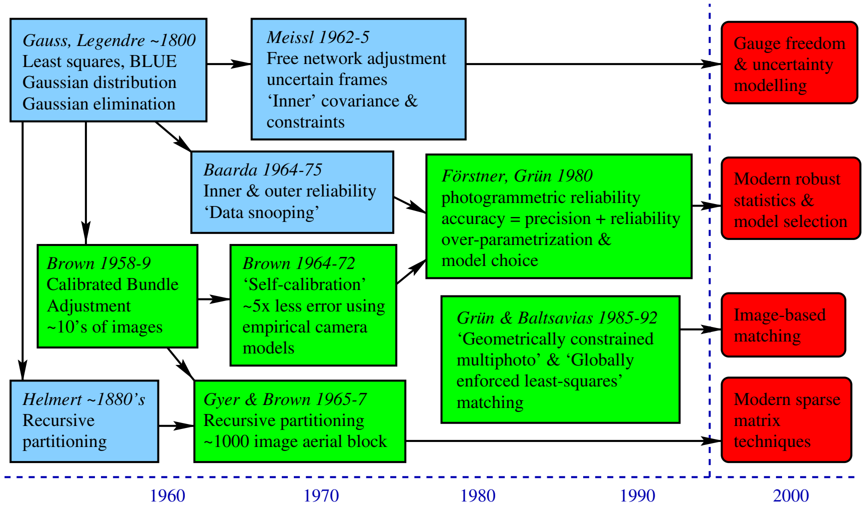
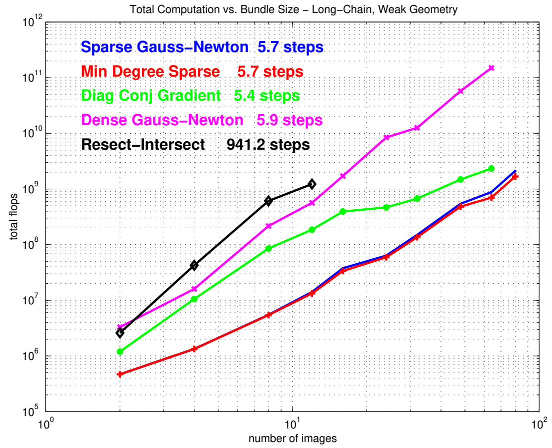
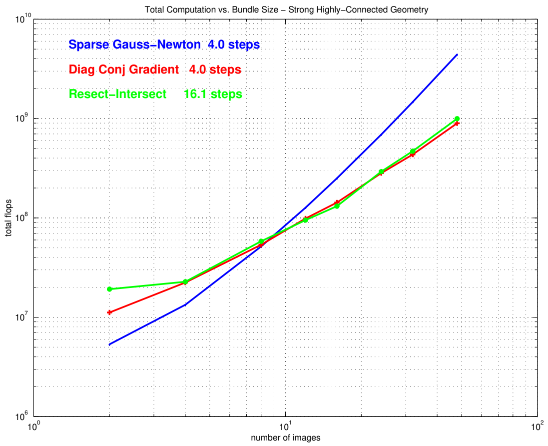
Bundle Adjustment - Schematic History





- Ripples from local corrections must propagate throughout image connection graph — methods that ignore/distort network (off-diagonal) information do this very slowly.
- For long chains like turntable sequences, all methods scale roughly as $\mathcal{O}(n^3)$, even "linear" ones.

Choice of Optimization Method



- Alternating resection and intersection is very simple and works quite well for strong, highly connected, low-perspective geometries . . .

  . . . but ***very badly*** for weak, loosely linked ones.

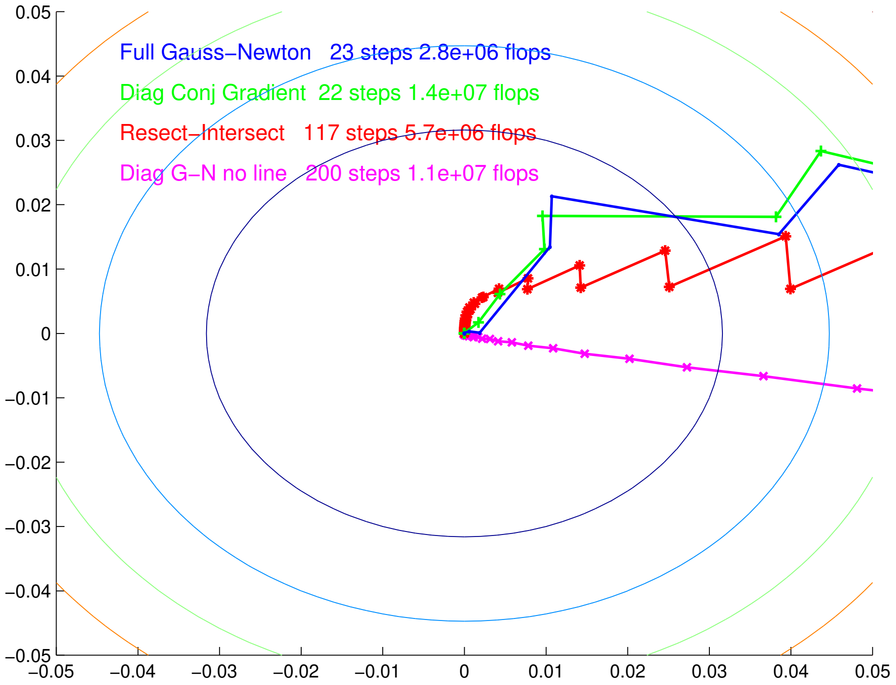
Why is Convergence So Slow ?

- If $\mathbf{x} = 0$ at the true solution, a Newton-like step estimate gives an iteration

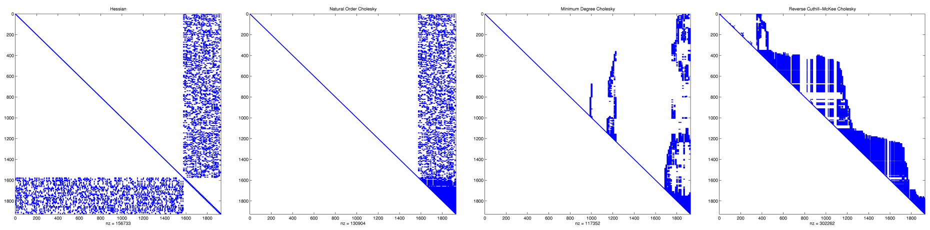
$$\mathbf{x}(i+1) \;=\; \mathbf{x}(i) - \mathbf{H}_{\text{approx}}^{-1}\, \mathbf{g}_{\text{true}}(i) \;\approx\; \left(\mathbf{I} - \mathbf{H}_{\text{approx}}^{-1}\, \mathbf{H}_{\text{true}}\right)\, \mathbf{x}(i)$$
$$\mathbf{g}_{\text{true}}\left(\mathbf{x}(i)\right) \;\approx\; \mathbf{H}_{\text{true}}\, \mathbf{x}(i)$$

Sandwich principle

- The iteration spends most of its time in the subspace spanned by the largest few positive and negative eigenvalues of $\mathbf{I} - \mathbf{H}_{\text{approx}}^{-1}\, \mathbf{H}_{\text{true}}$.
  - — It zig-zags for negative eigenvalues and crawls for positive ones.
  - — Adding a good line search helps a lot !
- Owing to gauge-freedom rank deficiency, dropping off-diagonal elements in $\mathbf{H}_{\text{approx}}$ *always* gives $\mathbf{I} - \mathbf{H}_{\text{approx}}^{-1}\, \mathbf{H}_{\text{true}}$ eigenvalues near ± 1
  - — *e.g.* $\mathbf{H}_{\text{true}} = \begin{pmatrix} 1+\delta & 1 \\ 1 & 1+\delta \end{pmatrix}$, $\mathbf{H}_{\text{approx}} = \begin{pmatrix} 1+\delta & 0 \\ 0 & 1+\delta \end{pmatrix}$ give eigenvalues $\pm\frac{1}{1+\delta}$.



Efficient Matrix Factorization



- The cost of matrix factorization depends strongly on the variable ordering.
- Most bundle implementations hardwire a single traditional ordering.
- Hardwiring is only practicable for a few simple, orderly configurations.
- Automatic heuristic variable orderings like minimum degree are often better than hand-built ones

— especially for site-reconstruction style bundles with complicated interconnections, constraints, prior information . . .