Autocalibration and the Absolute Quadric

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Absolute Quadric Ω

- A new projective encoding of ***Euclidean*** and ***affine*** structure
- Easier to use than traditional absolute conic + plane at infinity

Autocalibration

- New methods for ***autocalibration*** of moving camera with constant intrinsic parameters
- ***Rectify*** an initial projective reconstruction by enforcing ***absolute quadric projection constraint***

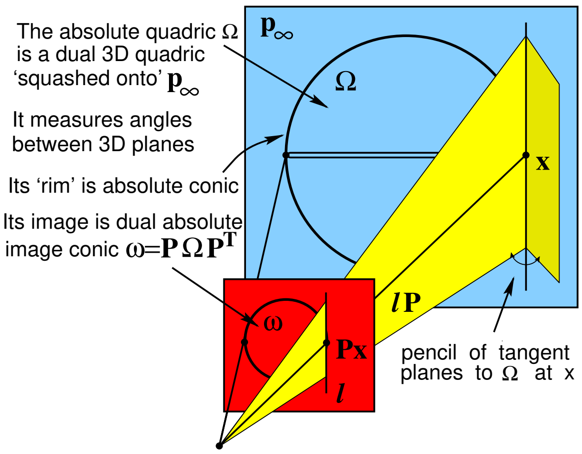
1. ***"Nonlinear"*** constrained numerical optimization method
   — much stabler than existing approaches
   — further constraints on calibration easily incorporated
2. ***"Quasi-linear"*** method from $m \geq 4$ images

General policy

- For precise results, stay close to the image data
- Avoid heavy algebraic elimination, *ad hoc* bases, . . .
- Resolve equations directly by numerical optimization

Absolute Quadric Ω

- A new projective encoding of ***metric*** + ***affine*** structure
- Dual to traditional absolute conic, but simpler to use
  — doesn't require separate encoding of plane at infinity
  — simple image projection
- In Euclidean coordinates $\Omega = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$
- In projective coordinates Ω becomes an arbitrary symmetric positive semidefinite contravariant 4×4 rank 3 matrix
- Abstractly, Ω is a degenerate dual 3D quadric 'squashed' onto plane at infinity. Its 'rim' is the absolute conic $\boldsymbol{C}$



Properties of Absolute Quadric

Angles between 3D planes — *i.e.* metric structure up to scale

$$\cos\theta_{12} = \boldsymbol{n}_1 \cdot \boldsymbol{n}_2 = (\boldsymbol{n}_1^\top \; d_1)\;\Omega\;\begin{pmatrix} \boldsymbol{n}_2 \\ d_2 \end{pmatrix}$$

Plane at infinity $\mathbf{p}_\infty$ — *i.e.* affine structure

$$\mathbf{p}_\infty \sim (\boldsymbol{0}\;1) \qquad \Longleftrightarrow \qquad \mathbf{p}_\infty\,\Omega = 0$$

Image Projection ω is ***dual of image of absolute conic***

$$\omega \;\equiv\; \boldsymbol{K}\boldsymbol{K}^\top \;\sim\; \mathbf{P}\,\Omega\,\mathbf{P}^\top$$

- $\boldsymbol{K}$ is ***internal camera calibration***

$$\mathbf{P} \;=\; \boldsymbol{K}\,(\boldsymbol{R}\,|\,\boldsymbol{t}) \qquad \text{where} \qquad \boldsymbol{K} \;\equiv\; \begin{pmatrix} \alpha_u & s & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{pmatrix}$$

- ω measures 3D angles between visual planes of image lines
- Can recover $\boldsymbol{K}$ from ω by ***Choleski decomposition***

Autocalibration

- Calibration and metric 3D structure from a camera with fixed unknown intrinsic parameters moving in an unknown scene
- Requires $m \geq 3$ images and ***generic motions***
- Partial information for non-generic motions:

camera preserves a "vertical" $\Rightarrow$ can't recover aspect ratios
pure translation $\Rightarrow$ affine 3D structure, but no calibration
pure rotation $\Rightarrow$ calibration, but no 3D structure

- Several existing approaches, but none very stable
  — heavy algebraic elimination $\Rightarrow$ poor numerical conditioning

Basic Principles

1. Uncalibrated cameras give only ***projective structure***
2. Metric structure $\Longleftrightarrow$ camera calibration
3. Use ***motion constancy*** of ***camera calibration*** to recover ***projective 'signature'*** of ***3D metric structure***
4. Use this to ***rectify*** an initial projective reconstruction

Approaches to Autocalibration

Kruppa Constraints

- Apply general two-image quadric matching constraints to absolute quadric

$$F_{21}\,\omega\,F_{21}^\top \;\sim\; [e_{12}]_\times\,\omega\,[e_{12}]_\times^\top$$

- Constraint says that measuring angles between epipolar planes in either image gives the same (3D) answer
- Cross-multiply to eliminate unknown scale factor

$\Rightarrow$ 2 independent quadratic constraints on ω given F, e

- Solve for ω from $m \geq 3$ images

(+) No explicit 3D reconstruction required

(−) Very sensitive to perturbations in e, F

(−) Recovered Ω could be *any* quadric with constant projection

Infinite Homography

- Images of distant points depend only on ***camera rotation***

$$|\text{distance}|/|\text{translation}| \;\gg\; (\text{focal length in pixels})$$

- ***Infinite homography*** H_∞ encodes rotation projectively

— estimate from distant-point correspondences

— can also use virtual (*e.g.* vanishing) points

- ***Rigidity constraint:*** $\omega \;\sim\; H_\infty\,\omega\,H_\infty^\top$

— recover ω linearly from rigidity and ≥ 2 observed H_∞'s

(+) Simple, direct and relatively stable

(−) Requires known-distant points, *i.e.* affine structure

— otherwise H_∞ can only be recovered indirectly via ω, Ω

Our Approach — Absolute Quadric Projection

- Initial ***projective reconstruction*** gives $\mathbf{P}_1, \ldots, \mathbf{P}_m$ up to

— individual rescaling $\mathbf{P}_i \to \lambda_i \mathbf{P}_i$

— overall 3D homography $\mathbf{P}_i \to \mathbf{P}_i \mathbf{T}$

- In this basis, Ω is unknown, 4×4 rank 3 matrix
- In each image, Ω projects to *same* dual absolute conic ω

$$\omega \;\sim\; \mathbf{P}_i\,\Omega\,\mathbf{P}_i^\top$$

- Cross-multiply to eliminate scales:

$$\omega^{AB}\left(\mathbf{P}_i \Omega \mathbf{P}_i^\top\right)^{CD} - \left(\mathbf{P}_i \Omega \mathbf{P}_i^\top\right)^{AB}\omega^{CD} = \mathbf{0}$$

$\Rightarrow$ $15m$ ***bilinear equations*** in 10+6 unknowns Ω, ω

- Write constraints symbolically as $\omega \wedge (\mathbf{P}\,\Omega\,\mathbf{P}^\top) = \mathbf{0}$
- Solve by ***constrained optimization*** or ***linearization***

— supplementary quartic constraint $\det(\Omega) = 0$

Solving for Ω, ω

Constrained Numerical Optimization

- Minimize $\sum_i \|\omega \wedge (\mathbf{P}_i \Omega \mathbf{P}_i^\top)\|^2$ given $\det(\Omega) = 0$
- ***Sequential quadratic programming (SQP)*** based algorithm

(+) Accurate — takes all constraints into account

(+) Additional constraints easily imposed

— *e.g.* known skew, aspect ratio, principal point ...

(−) Initial guess required — but not too critical

"Quasi-Linear" Method

Quadric projection equations $\omega \wedge (\mathbf{P}_i \Omega \mathbf{P}_i^\top) = \mathbf{0}$ give $15m$ ***linear*** constraints on $6 \cdot 10 = 60$ unknowns $\mathbf{x} = \Omega \otimes \omega$

1. Solve linearly for $\mathbf{x}$'s by homogeneous least squares (SVD)
2. Write $\mathbf{x}$'s as 6×10 matrix and project to rank 1 (SVD#2)
3. Extract ω and Ω, project Ω to rank 3 (SVD#3)

(+) No initialization required

(−) Requires $m \geq 4$ images as nonlinear constraints ignored

(−) Slower & less stable than SQP — fails for fixating camera

Sequential Quadratic Programming

- Iterative Newton-like method for ***constrained optimization***
- Minimize cost $f(\mathbf{x})$ given constraints $\mathbf{c}(\mathbf{x}) = \boldsymbol{0}$

1. ***Lagrange multipliers*** $\mathbf{z}$ give implicit solution:

$$\boxed{\nabla f + \mathbf{z} \cdot \nabla \mathbf{c} = \boldsymbol{0} \quad \text{with} \quad \mathbf{c}(\mathbf{x}) = \boldsymbol{0}}$$

2. Approximate cost & constraint to 2^{nd} order, to give exactly soluble ***constrained quadratic programming*** subproblem:

$$\boxed{\begin{pmatrix} \nabla^2 f & \nabla \mathbf{c}^\top \\ \nabla \mathbf{c} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \delta \mathbf{x} \\ \mathbf{z} \end{pmatrix} = -\begin{pmatrix} \nabla f \\ \mathbf{c} \end{pmatrix}}$$

3. Solve, update $\mathbf{x} \mathrel{+}= \delta \mathbf{x}$, iterate to convergence

(+) Quadratic asymptotic convergence

— *much* better than most *ad hoc* methods

(+) Numerically stable & accurate

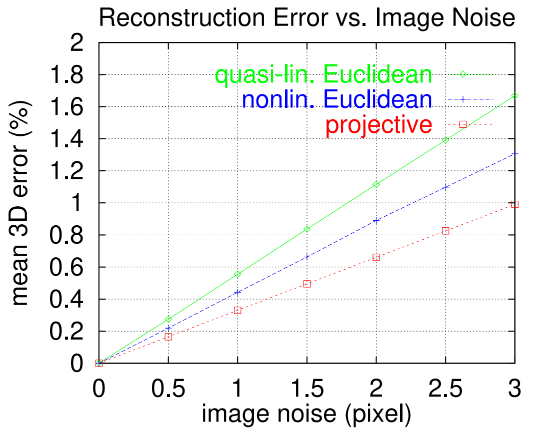
(+) Can handle arbitrary constraints

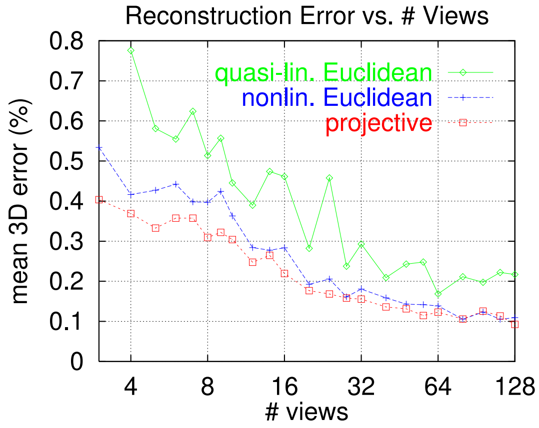
(−) Initialization required, may get stuck in local minima

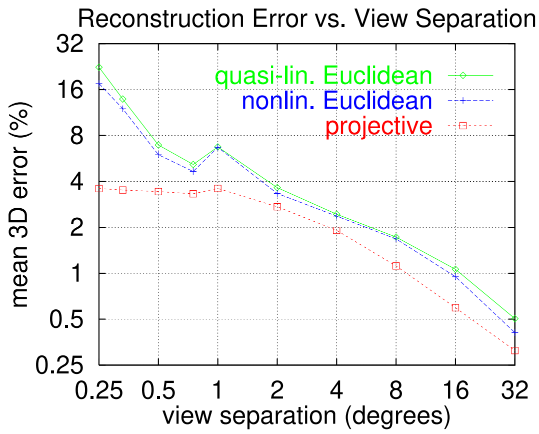
Autocalibration Algorithm

1. Normalize all image coordinates
2. Projective reconstruction $\Rightarrow$ projections $\mathbf{P}_i$
3. Minimize error $\sum_i \|\boldsymbol{\omega} \wedge \left(\mathbf{P}_i \Omega \mathbf{P}_i^\top\right)\|^2$ over $\boldsymbol{\omega}$ and Ω subject to $\det(\Omega) = 0$ (nonlinear & quasi-linear methods)
4. Choleski decomposition $\boldsymbol{\omega} = \boldsymbol{K}\boldsymbol{K}^\top \Rightarrow$ camera calibration $\boldsymbol{K}$
5. SVD Ω & extract rectifying transform $\mathbf{T}\,\Omega\,\mathbf{T}^\top = \left(\begin{smallmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & 0 \end{smallmatrix}\right)$
6. $\mathbf{P}_i \;\rightarrow\; \boldsymbol{K}^{-1}\,\mathbf{P}_i\,\mathbf{T}^{-1} \sim (\boldsymbol{R}_i \,|\, \boldsymbol{t}_i)$
7. *Optional:* adjust $\mathbf{P}_i$ to be exactly Euclidean and re-estimate 3D structure with new $\mathbf{P}_i$'s
8. *Optional:* bundle adjustment . . .

Experiments







Comparison with Conventional Calibration

- Informal test on precise calibration grid data
- All intrinsic parameters were within 1–2σ of conventional calibration
- Results may be more accurate than our precision calibration grid . . .

Summary

- The ***absolute quadric*** Ω

  — a simple projective representation of metric structure

- Autocalibration by ***absolute quadric projection*** constraint

  — gives stable, accurate & practical numerical method

Future Work

- Immediate extensions to

  — varying and/or constrained internal parameters

  — several moving cameras and stereo heads

- Automatic detection of degeneracy & constraint violations
- Specialized methods for small, continuous motions
- General SQP based approach to constrained geometric fitting