Factorization Methods for Projective Structure & Motion

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PROJECTIVE DEPTH + FACTORIZATION

$\Downarrow$

PROJECTIVE STRUCTURE + MOTION

- NEW "***Projective depth***" & ***depth recovery relations***
- NEW Factorization-based ***projective*** reconstruction [4]
  - ~~Affine~~ ***Fully Perspective*** camera model
- NEW ***Lines*** as well as points
- NEW Can also use ***trivalent tensor***

- ⊕ Very ***stable*** & ***accurate*** for points
- ⊕ All images & tokens treated equally
- ⊕ No initialization required
- ⊖ Points must be matched across all images
- ⊖ Requires epipolar (or tri-polar,…) geometry

Projective Depth

POINT PROJECTION (homogeneous coordinates):

$$\underbrace{\lambda}_{\text{`projective depth'}}\,\underbrace{\mathbf{x}}_{\text{image point}} = \underbrace{\mathbf{P}}_{\text{image projection}}\,\underbrace{\mathbf{X}}_{\text{world point}}$$

- For m images of n points:

$$\begin{pmatrix} \lambda_{11}\,\mathbf{x}_{11} & \cdots & \lambda_{1n}\,\mathbf{x}_{1n} \\ \vdots & \ddots & \vdots \\ \lambda_{m1}\,\mathbf{x}_{m1} & \cdots & \lambda_{mn}\,\mathbf{x}_{mn} \end{pmatrix} = \begin{pmatrix} \mathbf{P}_1 \\ \vdots \\ \mathbf{P}_m \end{pmatrix} \begin{pmatrix} \mathbf{X}_1 \cdots \mathbf{X}_n \end{pmatrix}$$

rank 4: factorize $\Longrightarrow$ motion + structure

PROJECTIVE DEPTHS

- ***Projective scale factors*** λ_{ip}
- The projective counterpart of ***optical depths***

  — reduce to optical depths in a Euclidean frame
- The same as Sparr's "***depths***" [3]
- Different from Shashua's "***projective depths***" [2]

  — these are cross ratios, not camera-relative depths

Joint Image Approach

PROJECTIVE DEPTHS

contain an

IMPLICIT PROJECTIVE RECONSTRUCTION

JOINT IMAGE

- Rescaled points lie in ***Joint Image*** [6]

— a 3D projective subspace of space of combined image coordinates

— a ***faithful projective copy*** of 3D world

DOF

- The mn depths λ_{ip} have $(m-1)(n-1)$ independent d.o.f.

— only defined modulo rescalings of projections & world points

i.e. rows and columns of rescaled data matrix

- ***Depth cross ratios*** $\frac{\lambda_{\rm ip}\lambda_{\rm jq}}{\lambda_{\rm jp}\lambda_{\rm iq}}$ are ***invariants of camera geometry***

Individual projective depths are arbitrary

BUT

Together they determine ***3D projective structure***!

Depth Recovery Relations

IMAGE POINTS + F-MATRICES + EPIPOLES

$\Downarrow$

PROJECTIVE DEPTHS

$$\boxed{(\mathbf{F}_{\mathrm{ij}}\, \mathbf{x}_{\mathrm{jp}})\, \lambda_{\mathrm{jp}} \;=\; (\mathbf{e}_{\mathrm{ji}} \wedge \mathbf{x}_{\mathrm{ip}})\, \lambda_{\mathrm{ip}}}$$

DEPTH RECOVERY

OLD Epipolar line passes through corresponding point & epipole

NEW ***Relative scale factor*** gives projective depths
i.e. $\lambda_{\mathrm{ip}}/\lambda_{\mathrm{jp}} \;\approx$ relative distance of points from epipoles

- Solve in least squares:

$$\boxed{\lambda_{\mathrm{jp}} \;=\; \frac{(\mathbf{F}_{\mathrm{ij}}\, \mathbf{x}_{\mathrm{jp}})\cdot(\mathbf{e}_{\mathrm{ji}} \wedge \mathbf{x}_{\mathrm{ip}})}{\|\mathbf{F}_{\mathrm{ij}}\, \mathbf{x}_{\mathrm{jp}}\|^2}\, \lambda_{\mathrm{ip}}}$$

DEPTH CHAINING

- For each point, propagate solution through all images

— parallel (key image) & serial (adjacent images) chains

— redundant chains also possible (consistent scaling needed!)

COMING SOON Analogous relations & reconstruction methods for:

— trivalent & quadrivalent tensors

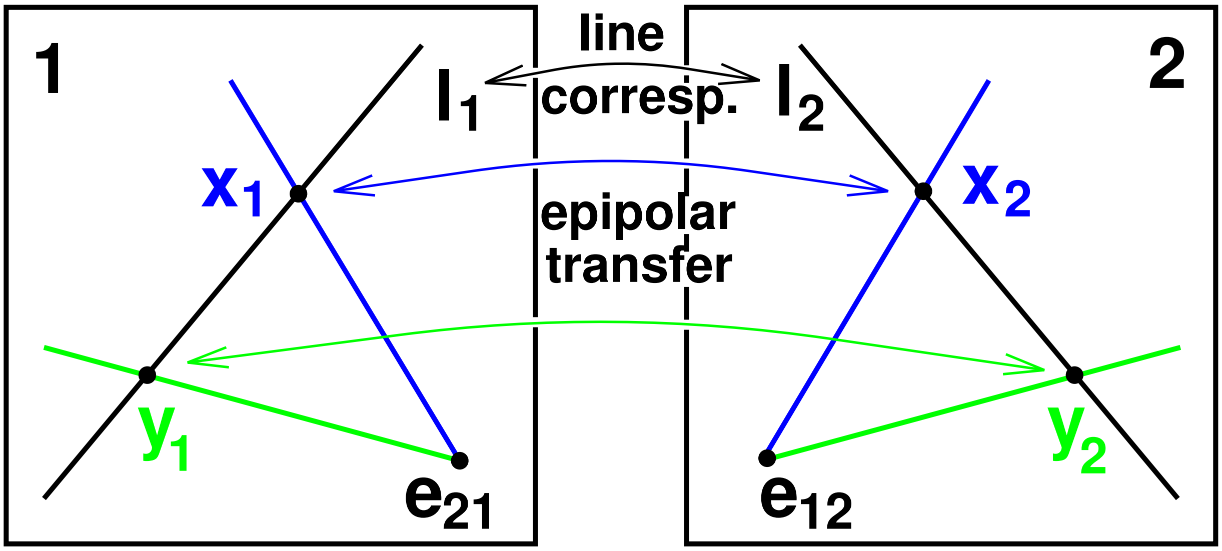
— projection matrices instead of points [7]

Line Reconstruction

1. Represent lines by ***pairs of via points***

2. Via point correspondence by ***epipolar transfer***

3. ***Reconstruct via points*** to find 3D lines



Via point correspondence by epipolar transfer

LINES + DEPTH RECOVERY RELATIONS

ONE $\Downarrow$ STEP

POINT TRANSFER + DEPTH RECOVERY

$$\mathbf{l}_i \wedge (\mathbf{F}_{ij}\, \mathbf{y}_j) \;=\; -(\mathbf{l}_i \cdot \mathbf{e}_{ji})\, \mathbf{y}_i$$

VIA POINT TRANSFER

NB

- Transfer is ***ill-conditioned*** for near-epipolar lines
- For good conditioning use ***widely spaced*** via points

Factorization

⊕ Concretizes ***reconstruction implicit in depths***

⊕ ***Reduces error*** by least-squares averaging/rank truncation

- ***Rescaled data matrix*** has ***rank 4 decomposition***

$$\begin{pmatrix} \lambda_{11}\,\mathbf{x}_{11} & \cdots & \lambda_{1n}\,\mathbf{x}_{1n} \\ \vdots & \ddots & \vdots \\ \lambda_{m1}\,\mathbf{x}_{m1} & \cdots & \lambda_{mn}\,\mathbf{x}_{mn} \end{pmatrix} = \begin{pmatrix} \mathbf{P}_1 \\ \vdots \\ \mathbf{P}_m \end{pmatrix} \begin{pmatrix} \mathbf{X}_1 \cdots \mathbf{X}_n \end{pmatrix}$$

- All such decompositions are ***equivalent***:

— freedom is 3D projective transform $\mathbf{X}_p \to \mathbf{T}\mathbf{X}_p, \mathbf{P}_i \to \mathbf{P}_i\mathbf{T}^{-1}$

SVD

- Factorize using ***Singular Value Decomposition***

— *c.f.* Tomasi-Kanade [5]

— very stable, but slow for large problems

FIXED RANK

- ***Fixed rank factorizations*** are $\mathcal{O}(\min(3m, n))$ faster

— linear in size of input data

— still quite stable

— iterative QR method (*c.f.* [1]) slightly more accurate than
1 sweep Gram-Schmidt method in paper

Algorithm

0 Extract & match points & lines across all images.

1 Standardize image coordinates to unit box.

2 Estimate chain of F-matrices & epipoles linking images.

3 ***Points:*** Estimate projective depths by chaining depth equations.
Lines: Choose via points & propagate through all images.

4 Build & balance* rescaled point matrix.

5 Factorize using SVD or fixed-rank method.

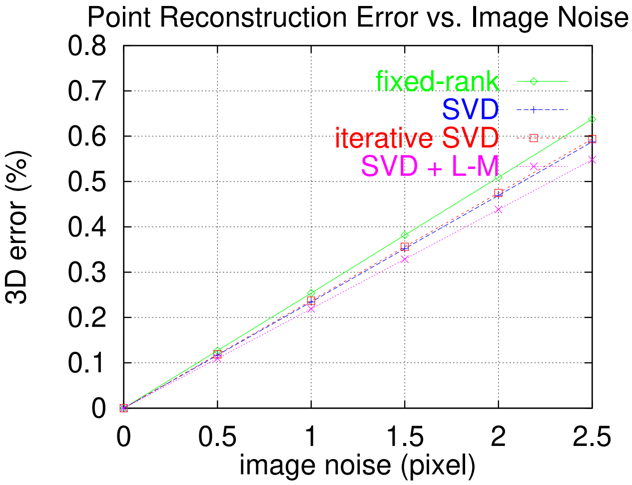
6 Read off 3D projective structure & motion.

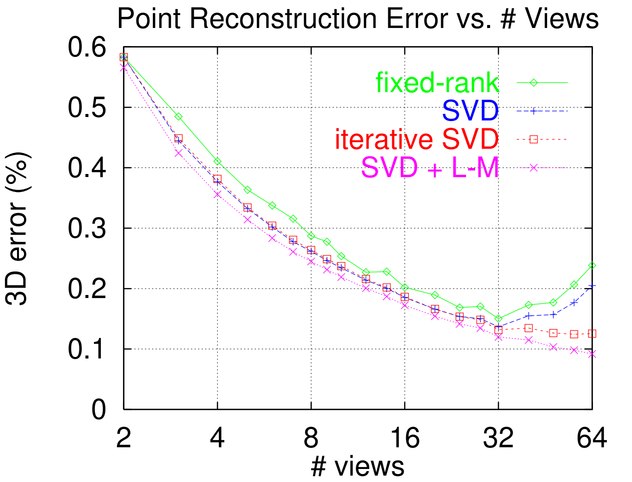
*Rescale rows & columns for good numerical conditioning.

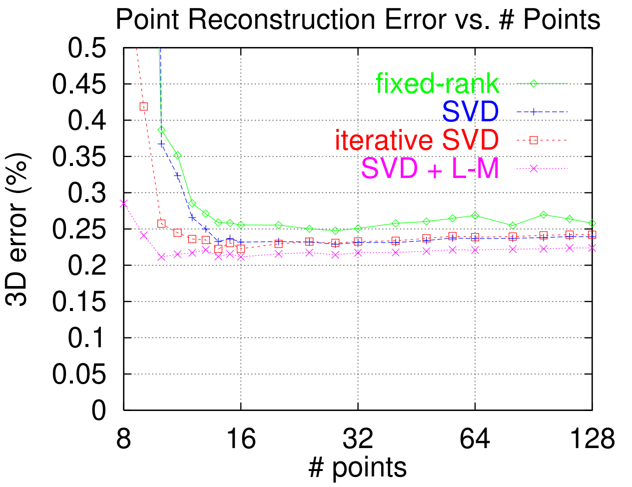
OPTIONAL Refine depths & via points by reprojecting estimated structure: iterate over steps 3–5.

Experiments

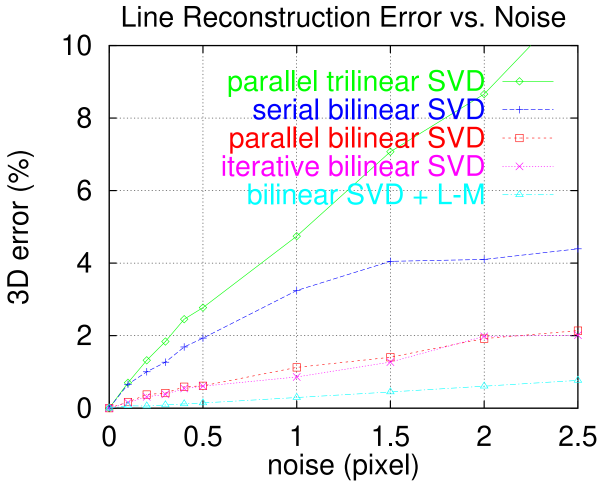
Points

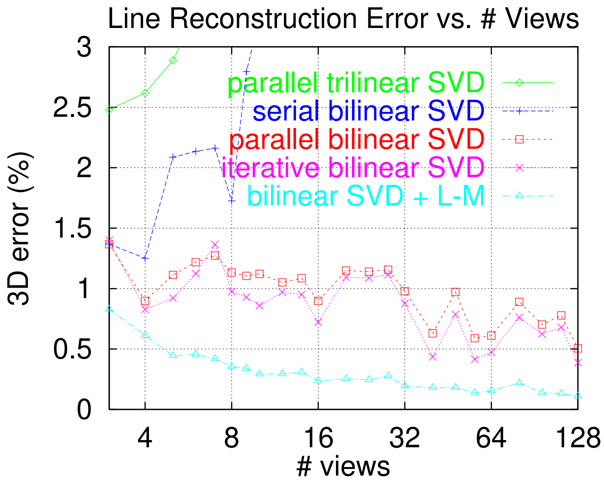


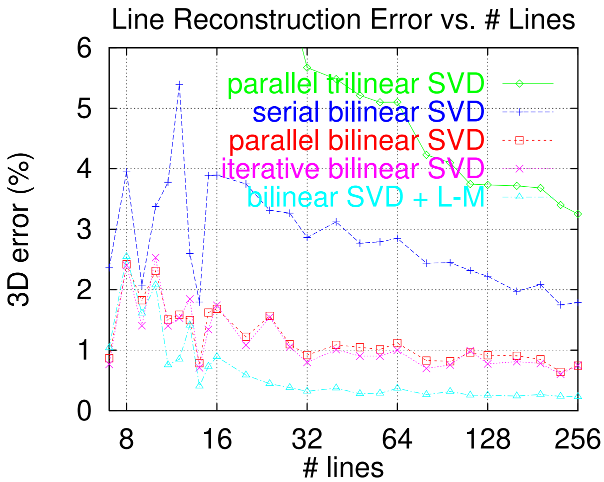




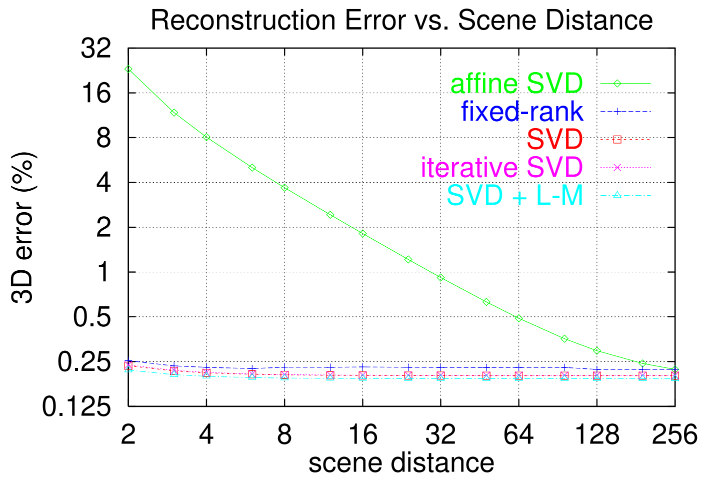
Lines



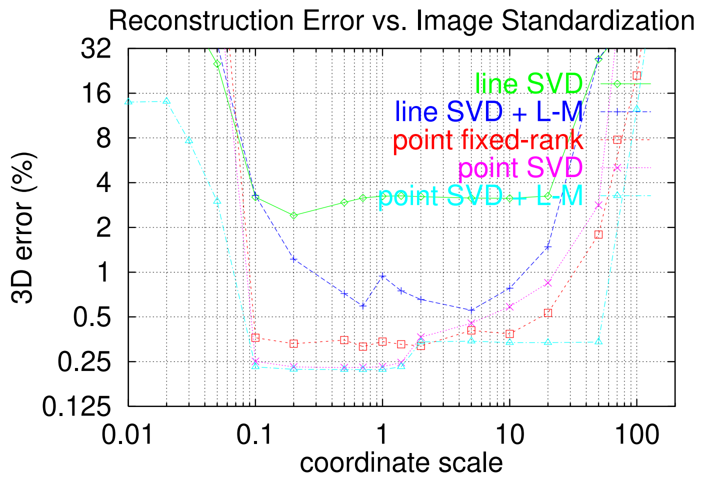




Synthetic data, wide angle lens, wide viewing angle



Affine *vs.* projective reconstruction error



Standardize *your* image coordinates!!

Conclusions & Future Work

⊕ Depth recovery for points ***extremely stable***
— even for chains of up to 50 images

⊕ Factorization-based point reconstruction very accurate & stable

⊖ Line reconstruction works, but much less stable
— intrinsically ill-conditioned problem, outliers common

COMING SOON!

- Real images!
- Redundant depth recovery equations
- Factorization with incomplete matches
- Trivalent tensor based methods

[1] T. Morita & T. Kanade. A sequential factorization method for recovering shape & motion from image streams. *IUW'94*.

[2] A. Shashua. Projective structure from uncalibrated images: Structure from motion & recognition. *PAMI* 16(8), 1994.

[3] G. Sparr. Depth computations from polyhedral images. *ECCV'92*.

[4] P. Sturm & B. Triggs A Factorization Based Algorithm for Multi-Image Projective Structure & Motion. *ECCV'96*.

[5] C. Tomasi & T. Kanade. Shape & motion from image streams under orthography: a factorization method. *IJCV*, 9(2), 1992.

[6] B. Triggs. Matching constraints & the joint image. *ICCV'95*.

[7] B. Triggs. Linear projective reconstruction from matching tensors. *BMVC'96*.

http://www.inrialpes.fr/movi/Triggs